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Article

The Duality of Psychological and Intrinsic Time in Artworks †

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Abstract: The paper investigates the problem of the time arrow in terms of artistic creation. The statistical model of artwork signal processing is established in order to identify a duality relation between psychological and intrinsic time. The intrinsic time is linked to the time operator of wavelets, and the psychological one is metaphorically related to the spatial domain of an artwork. The increase of irreducible randomness along the timeline is formulated by the second law of thermodynamics. The dual statement concerns an increase in statistical complexity, which is the definition of self-organization. In that manner, two arrows of time which are opposed and dual to each other are recognized. The authors have indicated a link to the theory of musical forms, the originality issue, and the perspective problem. Some repercussions for art theory, neuroaesthetics, and psychophysics have also been implied.

Keywords: statistical complexity; self-organization; time operator; wavelets; hidden Markov model; arrow of time; theory of musical forms

MSC: 00A65; 00A66; 37E05; 42C40

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1. Introduction

According to John Earman, the main issue concerning "the problem of the time arrow" is to figure out what the problem is supposed to be [1]. The quoted phrase has originated from Arthur Eddington who inferred a great thing about time: that it goes on [2]. The paper aims to elucidate the implications of this inference in terms of artistic creativity, wherein the time flow is an intrinsic component.

In Eddington's view, the flow of time is the condition of consciousness which provides a straightforward insight into becoming. However, the fundamental physics by no means corresponds to any indication of time flow [2]. Physical time is arrowless, and it is consciousness that declares and detects the arrow of time. Such an attitude is rather one-sided, but it is quite widespread in the scientific community. Without delving into the causes and consequences of this worldview, it is important to note that the stark separation between physics and consciousness is unproductive for an elaboration of the topic. On the contrary, in the effort to bridge these domains of experience, time appears to have assumed a central role [3].

The issue was elaborated by the Brussels School of Thermodynamics which concerned an extension of fundamental physics in order to include phenomena implying the time arrow. Considering a system in terms of the Koopman formalism, one has postulated the intrinsic time operator which indicates an irreversible and non-local behavior [4]. The existence of irreversibility in the physical theory based upon reversible dynamics represents an incompleteness statement, resembling the Gödel theorem in arithmetic [5]. Such a complex system view should suppose both deterministic and stochastic processes to be fundamental, since randomness entered theory through the back door [6].

Complex systemsrequire a measure of complexity which is dual to entropy and fractal dimensions [7]. The statistical complexity, suggested by Grassberger, appears to be a satisfactory solution [8]. Its increase in the temporal domain represents self-organization, which might be seen as a dual statement to the entropy increase [9]. According to Eddington, the increase in entropy defines the time arrow due to the second law of thermodynamics. On the other hand, the complexity increase has defined an arrow of time which relates to organization and creativity. In that regard, two concepts of time dual to each other are recognized.

It is already demonstrated that self-organization corresponds to art creation as well as its observation [10]. The method is based upon an effective algorithm for quantifying complexity using wavelet decomposition [11]. It implies a statistical model of artwork signal processing wherein the intrinsic time corresponds to the scale [12]. The psychological time, which is an alternative concept, should be presented in terms of the duality interrelation. Dual concepts originate from Étienne Souriau, who has considered time in the plastic arts [13]. The authors are going to pay particular attention to such a consideration in order to indicate some repercussions for art theory, neuroaesthetics and psychophysics. A substantial relation to the measurement problem has been a crucial contribution of the model [14].

After the Introduction, Section 2 contains materials and methods applicable to the spatio-temporal geometry of artworks. Particular attention should be paid to dual concepts of psychological and intrinsic time by Souriau who moreover claimed that the study of musical space was yet to be undertaken. In Section 3, the authors present a statistical model of artwork signal processing aimed to elucidate the duality interrelation. The topic is elaborated for an instance of musical signals due to the existence of the default timeline whose expansion pervades a sensation of temporality. A comprehensive discussion of the model is given in the next section involving the theory of musical forms and the originality issue. The last section contains concluding remarks.

2. Materials and Methods

2.1. Psychological and Intrinsic Time

Pavel Florensky regarded art as a process of spatial organization [15] (XII, p. 48). The creative process concerns an artist's ability to transfigure realitywhich is spatially organized. Art is therefore an attempt to reorganize space, arranging it in a self-manner. This definition indicates self-organization of complex systems, which implies a complexity increase in the temporal domain [16]. Florensky was followed by Clemena Antonovaasserting that "time is a fundamental organizing principle of pictorial art" [17] (p. 5).

Antonova referred to the article by Étienne Souriau discussing time in artworks that principally concerns plastic arts, though he suggested as well a generalized approach to different art forms [13]. At the very beginning, Souriau states: "Nothing is more dangerous for exact and delicate understanding of the plastic arts (design, painting, sculpture, architecture, and minor arts) than their rather banal description 'arts of space,' in contrast to the phonetic and cinematic arts (music, poetry, the dance, and to this group we must now add the cinema), characterized as 'arts of time'.

> This contrast, perceived by many aestheticians, from Hegel to Max Dessoir, has its historic origin in the philosophy of Kant, particularly in the contrast he makes between the external senses, to which the form of space would be inherent, and the internal sense whose form would be time. The desire to bring music and poetry into the realm of the internal sense (in order to see there 'the soul speaking directly to the soul') has often led to a real misunderstanding of the extent and the cosmic reach of the plastic arts, stripped of their temporal dimensions, and of their content according to that dimension".

The ill-founded contrast between arts of space and arts of time is based upon the fact that the physical frame at first ones remains materially unchanging. But the same conclusion should emerge considering extensional frame of any art:

> "The disc on which a musical composition is recorded also remains materially unchanging. The disc, however, is but an instrument for the orderly presentation of the work which itself is the structural law of the latter, and which governs the musical execution. One must see in the same way the movement of the spectator around the statue or the architectural monument as a plastic or view-absorbing execution, which unfolds in order the various aspects which are held within the physical frame, and which are the aesthetic reason for that frame as it was planned".

The arising concept is termed by Souriau "psychological time". It points out the spectator's involution in the perception which is subjected to an extensional frame.

Another mode of temporality, termed "intrinsic time", corresponds to the observation that is a process of reorganizing the frame [18]. Souriau defines it as "an artistic time inherent in the texture itself of a picture in its composition, in its aesthetic arrangement". In that respect, he indicates temporality which is not merely psychological but a compositional and aesthetical conception. However, the analysis of illustrations offered to substantiate this fails to show what exactly constitutes intrinsic time [19] (p. 191). Still, Souriau's article seems important pointing in a direction that should lead to fruitful results [17] (p. 10).

2.2. Spatio-Temporal Geometry of Icon

Composition deals with the elements of an artwork, whilst construction is a structuring principle that should define its integrity. Both of them are indispensable constituents of art. Composition is indifferent to the intensional signification of an artwork, considering only the extensional frame. Construction, on the contrary, is intensionally oriented and indifferent to such an extension. In a harmonious artwork, both of them are unified with no outweighting each other, and Florensky points to Hellenistic art and traditional iconography as particular instances of harmonized unity [15] (LXII–LXIII, pp. 94–98).

The integrity of an artwork is primarily due to its construction in an effort to transcend the elements that make up its composition. Elements should therefore be observed and evaluated in a different manner than if they were particularly perceived. According to the law of illusion which claims that "Elements appear different than they really are" [15] (III, pp. 238–251), reality corresponds to an extensional frame composed of specific elements which that are reorganized by an artwork. Preobrazhensky clarified it by stating: "A specific element seems to undergo a change opposite to being absorbed by an extensional frame" [20]. He has also outlined several rules that the illusion must adhere:

- To enhance the illusion, one should amplify the overall influence, composing an extensional frame, so that the specific impression made particularly is not weakened.
- The illusion does not vanish with careful observation nor does it increase.
- The illusion does not disappear even if we know its cause.

Self-organization, that concerns art creation and its observation, comes to be a play rite (Latin illudere—to play off) which is implemented as well by science [21]. It corresponds to the truth criterion, whose feature is an arrow of time defined via the complexity increase [10]. A ritual time closely intertwined with traditional iconography has implemented such a conception [22]. The point is quite obvious in comparison to the modern painting whose composition has prevailed over construction. In that regard, the image narrows by a linear perspective representing the extensional frame. On the opposite, an icon has spread into the reverse perspective which is spatio-temporal geometry that might be a paradigm of the harmonious unity between composition and construction (Figure 1). Such a canon stems from ancient frescoes which "tended to produce an illusory effect of light and materials" [23]. Hellenic art and traditional iconography are therefore particular instances of the same canon, which has supported Florensky's view [15] (LXIII, p. 98).

The depth of the reverse perspective corresponds to the intrinsic time of an artwork, which is a feature evident as well in the Icon of Christ's Transfiguration. An expansion of mountain massifs toward the image interior has established a progression from horizontal through semi-vertical to finally the vertical position with respect to the central Christ figure

(Figure 2). In that regard, iconography is considered to be transfigurative art representing a process which implies the time arrow. Self-organization is manifested by the emergence of the Von Koch curve whose scaling properties indicate a fundamental principle of the artwork organization which is connatural to fractal geometry. Fractals are chimerical forms which involve time grasped through a scale that has arisen from the construction of a figure, and similarity to the iconography is striking [24]. Although not explicitly discernible in each artwork, the instance raises a possibility to establish the statistical model.

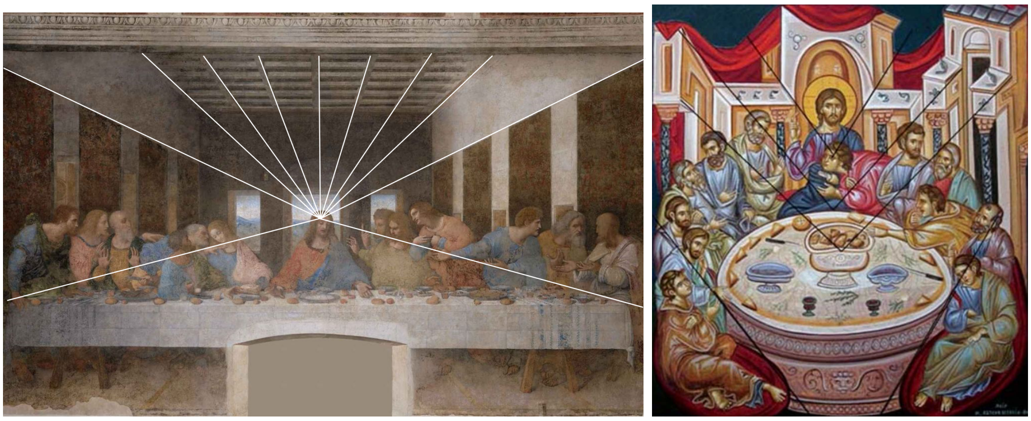

Figure 1. *The Last Supper*. (**left**) The modern painting by Leonardo da Vinci [25]. Christ occurs in the center of the image, where rays of the linear perspective intersect. (**right**) The traditional icon. Rays of the reverse perspective emerge out from the image, contextualizing the observer's personage position (vacant place near the table) [24].

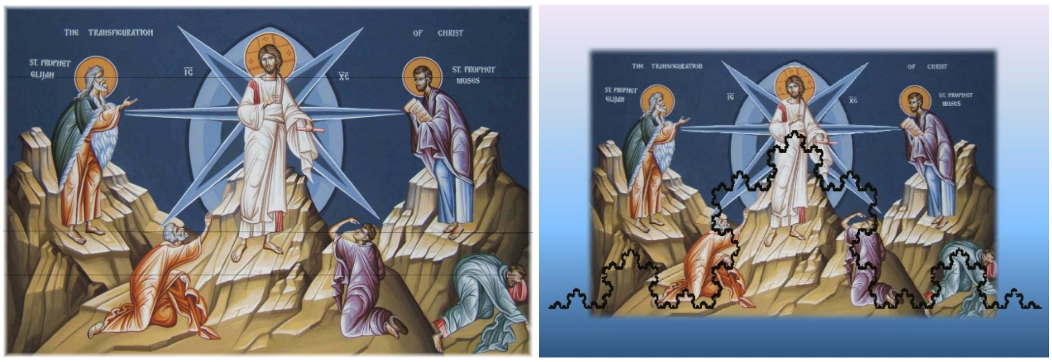

Figure 2. The Icon of Christ's Transfiguration. (**left**) The expansion of the mountain massifs has established progression from horizontal through semi-vertical to finally the vertical position with respect to the central Christ figure. (**right**) Scaling properties of the image disclosed by an emergence of the Von Koch curve [24].

The same principle pervades biology wherein a scale is related to the growth of an organism, which is perceived in tree branching [26]. Such a principle is therefore pervasive in geometry and biology as well as in physics through the intrinsic time operator defining complex systems [3]. The issue has been elaborated by the Brussels School of Thermodynamics, but it acutally incorporates the view on time by Bergson and some aspects by Huserl and Heidegger [27]. Bergson's view has been explicated in opposition to Einstein, who was supporting an objective epistemology of science [28]. The conception substantially relates to the time continuum by Brouwer that is a forerummer of postmodernism in science [29]. It completely fits the concept of vertical time by Bachelard, which has primarily applied to poetry [12]. An implication of the neuroaesthetics and psychophysics has already been discussed [10]. There is also a relation to concepts of observation and measurement in physics [14].

3. Results

3.1. Complex System Model

In formulating the artwork signal processing model, the authors have focused on the Hilbert space $L^2(\mathbb{I})$, which consists of one-dimensional signals on the unit interval. Extending the model to higher dimensions is straightforward, as the space of 2D signals is factorized into 1D through the tensor product. The 1D confinement is paradigmatically applied to musical signals, where the default timeline encompasses the psychological duration from the beginning to the conclusion of the artwork. In that regard, an extensional frame corresponds to the spatial domain $\mathbb{I} = [0,1]$, which is metaphorically related to temporality [30].

According to the methodological insight represented in Figure 2, a signal should be considered with respect to the scale, which relates to the intrinsic time. One requires therefore a multiscale representation of the Hilbert space, which is implemented by wavelet decomposition $f = A + \sum_{j\geq 0}\sum_{k=1}^{2^j} D_{j,k}\psi_{j,k}$ wherein A denotes an approximation coefficient and $D_{j,k}$ is a pyramid of detail coefficients. Each wavelet $\psi_{j,k}$ at a scale j is inherited by two of them sharing its support at the next one $j+1$, which makes the pyramid a binary tree (Figure 3). The intrinsic time $T\psi_{j,k} = j\psi_{j,k}$, that has multiplied a wavelet $\psi_{j,k}$ by the eigenvalue j corresponding to its scale, operates on a dense subset of $L^2(\mathbb{I}) \ominus \mathbb{1}$, which is the Hilbert space reduced by constant signals.

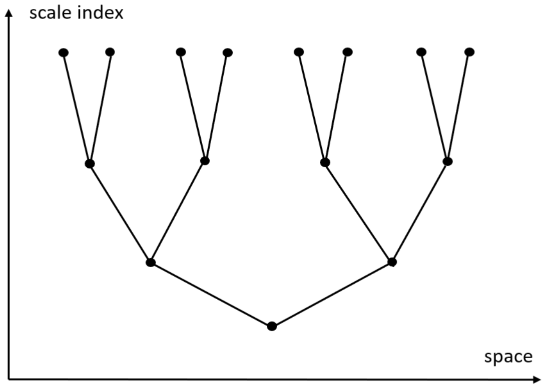

Figure 3. The tree of detail coefficients. Each wavelet at a scale is inherited by two of them sharing its support at the next one.

An expansion of the timeline which infuses a sensation of temporality is presented by the Rényi map $R\tau = \begin{cases} 2\tau, & 0 \leq \tau < \frac{1}{2} \\ 2\tau - 1, & \frac{1}{2} \leq \tau \leq 1 \end{cases}$ that should expand both halves of the unit interval onto an entire one. The operator induced $U : f(\cdot) \mapsto f(R\cdot)$ governs the evolution of detailed subspaces Δ_j, each of which is generated by wavelets $\psi_{j,k}$ at a certain scale j. The evolutionary axiom $f \in \Delta_j \iff Uf \in \Delta_{j+1}$ holds, which is equivalent to a wandering property $\Delta_j = U^{-1}\Delta_{j+1}$ that concerns the inverse image of a subspace. It is followed by the commutator relation

$$[T, U]\psi_{j,k} = TU\psi_{j,k} - UT\psi_{j,k} = (j+1)U\psi_{j,k} - jU\psi_{j,k} = U\psi_{j,k} \Longrightarrow [T,\ U] = U$$

which is a definition of the time operator [31].

The evolution by U is non-invertible, but it might extend to a unitary operator [32]. To that end, the Hilbert space $L^2(\mathbb{I})$ is extended into $L^2(\mathbb{I} \times \mathbb{I}) = L^2(\mathbb{I}) \otimes L^2(\mathbb{I})$ which provides a decomposition identity $F = A \otimes 1 + \sum_{j\geq 0}\sum_{k=1}^{2^j} D_{j,k} \otimes \psi_{j,k}$. Detailed coefficients of the binary tree $\mathbf{D} = \left(D_{j,k}\right)$ are regarded as random variables, as well as wavelets $\psi_{j,k}$, which is an indispensable term of the statistical model elaborated in Appendix A. In that respect, the operator U governs the evolution of variables on the domain $\mathbb{I} \times \mathbb{I}$, and distribution densities are governed by the adjoint operator $U^{\dagger} = U^{-1}$.

The extension also applies to the intrinsic time T, and the commutator relation holds. The time operator has induced a change in representation $\Lambda = \lambda(T)$ which transfigures the evolutionary group generated by $U^{\dagger}$ to a Markov semigroup by $W^{\dagger} = \Lambda U^{\dagger} \Lambda^{-1}$. Although there is the inverse operator $W^{-1\dagger} = \Lambda U \Lambda^{-1}$, it does not preserve positivity $\rho > 0 \not\Rightarrow W^{-1\dagger}\rho > 0$ and therefore should not map a density to another [33].

Such a change in representation transfigures $\mathbf{D} = \left(D_{j,k}\right)$ into a Markovian tree $\mathbf{S} = \left(S_{j,k}\right)$ of hidden variables which have exhausted the correlation between the detail coefficients (Figure 4). In that manner, the wavelet-domain hidden Markov model is established, which has is proven to be tremendously useful in a variety of applications including speech recognition and artificial intelligence [34]. The information contained in the detail coefficients is decomposed according to the equality $H(\mathbf{D}) = H(\mathbf{S}) + H(\mathbf{D}|\mathbf{S})$ wherein the first term is related to complexity and the second one is an irreducible randomness which corresponds to the free entropy. The global complexity $H(\mathbf{S})$ measures an increase in the local complexity $H\left(S_{j,k}\right)$, which is a definition of self-organization. In that respect, an arrow concerning the intrinsic time has been established [11].

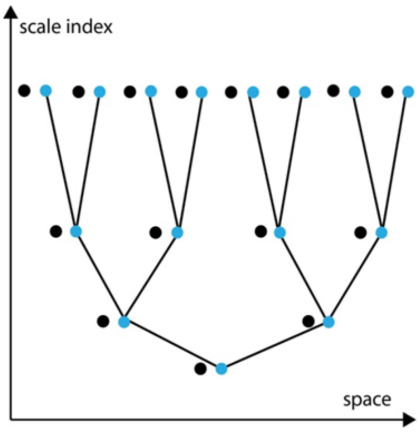

Figure 4. The wavelet-domain hidden Markov model. Black nodes correspond to detail coefficients and blue ones to hidden variables.

3.2. Duality of Times

The evolutionary operator U expands the timeline corresponding to the psychological time of the artwork. Such an expansion concerns stepping backward in the intrinsic time which is a scale of wavelet decomposition (Figure 5). In that manner, the complexity $H(\mathbf{S})$ has decreased since the intrinsic time is the axis of self-organization. On the other hand, information $H(U\mathbf{D}) = H(\mathbf{D}) + \log|\det U| = H(\mathbf{D})$ has remained the same, considering that U is a unitary operator. It follows from the equality $H(\mathbf{D}) = H(\mathbf{S}) + H(\mathbf{D}|\mathbf{S})$ that the irreducible randomness $H(\mathbf{D}|\mathbf{S})$ has increased along the timeline, which is a formulation of the second law.

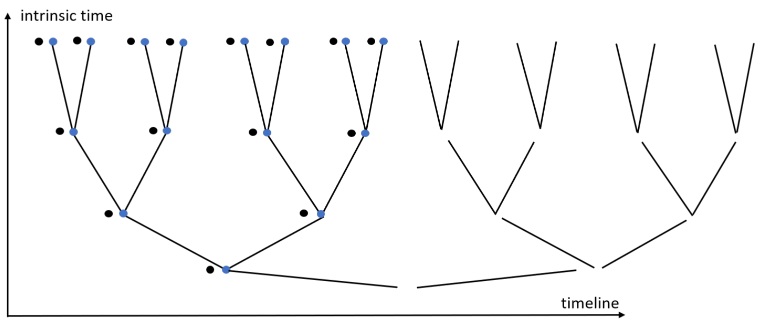

Figure 5. Expansion of the timeline stepping backwards in the intrinsic time. (**left**) The wavelet domain hidden Markov model. (**right**) The expansion involving one more scale.

In order to establish the dual statement, one should present a relation between the psychological time τ and the intrinsic one t. Let τ correspond to a part of the timeline which has already been seen by an observer, and let t correspond to the top of the subtree representing such a part. It is not hard to link them like $\tau = 2^{-t}$, or to be more accurate, $t = \lfloor \log_{\frac{1}{2}} \tau \rfloor$ considering that the intrinsic time has been bouncily defined. Anyway, the increase in t concerns the decrease in τ and therefore the increase in $H(\mathbf{S})$ and decrease in $H(\mathbf{D}|\mathbf{S})$. In that manner, two arrows emerge which are opposed and dual to each other.

The duality is presented by the spiral wherein the psychological time τ corresponds to the length and the intrinsic one t to the scale (Figure 6). Expansion of the timeline, due to the evolution by U, makes one more twist of the spiral, which is related to the wandering of detail subspaces $\Delta_j = U^{-1}\Delta_{j+1}$. The same property might be phrased in the manner of $\Delta_j = U^{\dagger}\Delta_{j+1}$, wherein $U^{\dagger} : f(\cdot) \mapsto \frac{f(\frac{\cdot}{2})+f(\frac{\cdot+1}{2})}{2}$ is the adjoint operator such that $U^{\dagger}U = I$. The evolution by $U^{\dagger}$ concerns a part of the artwork which has not been seen yet by an observer (Figure 7). It is termed "ontological time" $\theta = 1 - 2^{-t}$ that increases as well as t, which has established an alternative spiral (Figure 8).

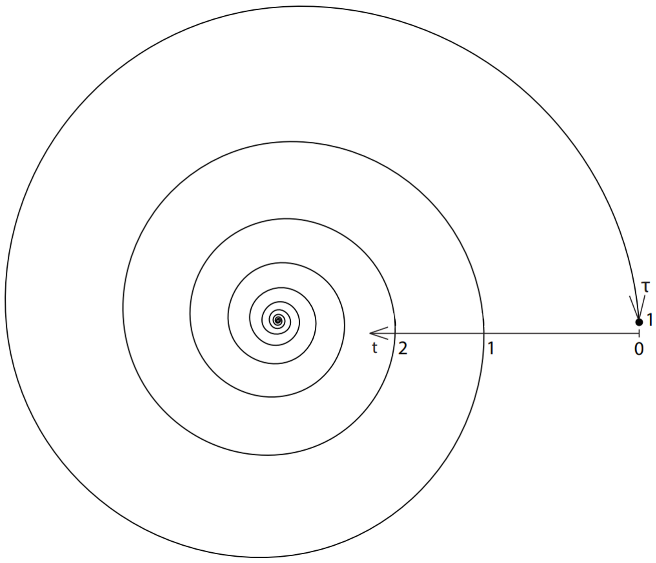

Figure 6. The time spiral representing the duality relation. The intrinsic time corresponds to the scale and the intrinsic one to the length of the spiral.

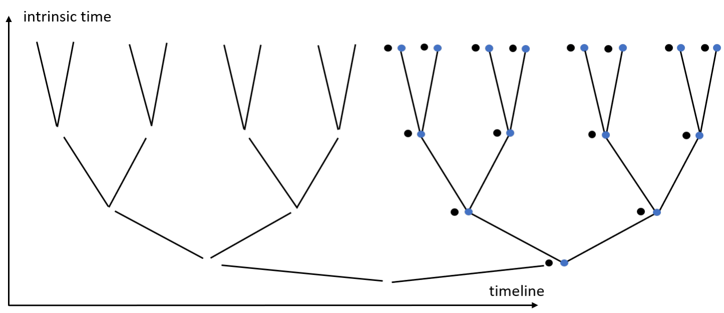

Figure 7. Contraction of the timeline. The ontological time concerns a part of the artwork which has not been observed yet.

Wavelets evolve via the axiom $U^{\dagger}\psi_i = \frac{1}{\sqrt{2}}\psi_{Ri}$, wherein $i = \frac{2k-1}{2^{j+1}}$ is a dyadic fraction which corresponds to the pair (j,k). Dyadic numbers are not ordered linearly but in regard to the norm $|i|_2 = 2^{j+1}$ following turns of the spiral. Each detail subspace is therefore related to such a turn which contains numbers indexing wavelets at the scale. The ontological time suits a value which has ended the turn.

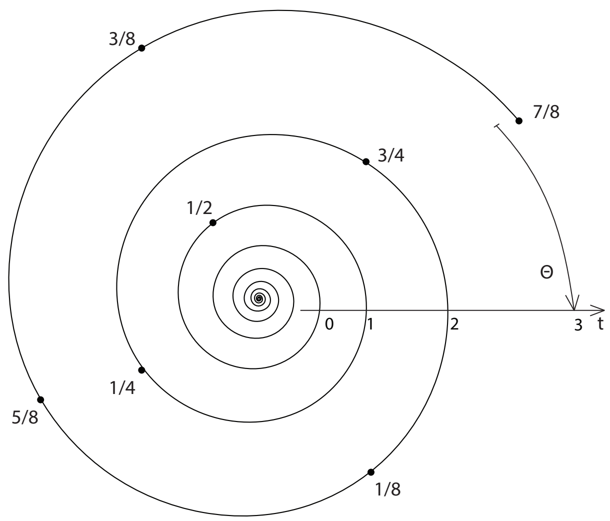

Figure 8. An alternative spiral of time corresponding to dyadic numbers. The ontological time suits a value that has ended the turn.

4. Discussion

4.1. Theory of Musical Forms

The model presented aligns with the foundational principles of the theory of musical forms established by Adolf Bernhard Marx in the first half of the 19th century. It is emphasized that these principles have emerged from practical experience, establishing a significant link to the creative process [35] (p. 29). A pivotal aspect lies in the insight that the theory of musical forms should not merely dictate form but rather focus on the cognition of information processing. Disregarding this fact could result in a theory that is nothing but “a compilation of departed models” [36] (p. 90, 96). While the form itself might lack inherent value, it stands as a criterion having the ultimate potential to involve aesthetic diversity.

Marx claims that the integrity of a musical piece is inviolable and “each separation is an abstraction which the art has no idea about”. In that regard, the form coincides to the construction of an artwork, harmonizing with the composition through self-organization. It follows that the form is a manner in which the substance emerges: “The form of an artwork is precisely and clearly denoted to be a manifestation of the substance” [36] (pp. 88–89). The concept has been further explored by other music theorists, including Arnold Schoenberg, who examined the form in relation to the creative process, such as Marx [37].

It is worth noting the analysis of the form through the examination and interpretation of the musical flow by Berislav Popović, which significantly relates to original principles of the theory. Popović has suggested a shift in focus from analyzing the formal pattern to examining the time flow, i.e., emergence of the musical substance. This analysis focuses on interpreting the piece of music as an integral organization, which is a cornerstone deeply embedded in the fundamental assumptions of the theory. A crucial principle of the methodology by Marx is formulated in a viewpoint that “the form is almost synonymous to integrity” [38]. On the other hand, Popović defined the musical flow to be the “integrity of the system whose parts relate to each other by dynamical interdependence” [39] (p. 15), which has identified form and flow.

He morevover argued that the concept of symmetry, which pertains to the construction of a form, should occupy a central position in realizing the coherence of a musical piece. Acknowledging symmetry as a crucial aspect of analysis requires a connection between geometry and music. Its manifestation in the musical flow has emerged as a critical factor, prompting the need for a precise definition of the phenomenon [35] (pp. 15–23). It is significant to recognize that the analytical procedure is just as creative as the process that gave rise to the form. The methodology, rooted in the concept of symmetry, naturally follows to be the key of identifying the originality, integrity, and excellence of an artwork. In that regard, a creation and the analytical interpretation become united by an artistic program which makes "not only music to be considered an analysis, but the analysis to be considered music as well" [40].

Symmetry represents the substance of geometry that allows one to establish a criterion which is applicable under general terms [10]. The concept has originally related to measurement (in Greek $\sigma\upsilon\mu\mu\epsilon\tau\rho\acute{\iota}\alpha$—commensuration), which is a process taking place step by step over time, via the Euclidean algorithm

$$\frac{a}{b} = \cfrac{1}{n_1 + \cfrac{1}{n_2 + \cfrac{1}{\ddots}}}$$

for magnitudes $a \leq b$. The question mark function by Minkowski $? : \cfrac{1}{n_1 + \cfrac{1}{n_2 + \cfrac{1}{\ddots}}} \mapsto \frac{1}{2^{n_1 - 1}} - \frac{1}{2^{n_1 + n_2 - 1}} + \ldots$ is an automorphism transfiguring continued fraction to binary code $\tau = \underbrace{0.\ \ldots}_{n_1}\underbrace{1\ \ldots}_{n_2}\underbrace{0\ \ldots}\,1\ \ldots$ wherein each digit corresponds to a step of the process [32]. The time flow concerns a shift leftwards in terms of binary digits, which has defined the Rényi map on the unit interval $\mathbb{I} = [0,\ 1]$. It is a symmetry as well according to the extensional definition by Felix Klein, which implies that a frame is invariant under the transformation. The spatio-temporal geometry which is postulated in that manner unifies the construction and composition of an artwork.

4.2. Originality of the Creative Process

The aesthetical criterion identifying a geometrical substance corresponds to the original creation in art. It relates self-organization whose defining feature is an arrow of time representing the complexity increase [10]. The model has been corroborated by experimental results concerning the originality issue. The data set consisting of seven high-resolution images painted by the Dutch artist Caspers has been used. The artist was commissioned by Ingrid Daubechies and the members of the Machine Learning and Image Processing for Art Investigation Research Group at Princeton University to paint seven paintings of relatively small size (approximately 25 cm $\times$ 25 cm) by different styles and using different materials [41].

Within the next few days, she has also produced a replica of each painting, using the same paints, brushes and grounds and under the same lighting conditions (Figure 9). For the model and results, it is of interest to mention the remark of Daubechies that Caspers spent nearly twice as much time on creating each replica as compared to the original, which indicates that "painting a copy is a more painstaking process than the spontaneous painting of an original". The copies were of such exceptional quality that the artist was convinced no observer might be able to distinguish them from the originals. However, it has been demonstrated that original paintings have higher complexities than replicas of the same [42].

In the statistical model, reproducing an artwork is related to expansion of the extensional frame due to the Rényi map. It is evident in the fact that the psychological time has doubled up, which coincides to going a step backward in the intrinsic one. Consequently, replicas exhibit decreased complexity, as the arrow opposes self-organization. In such an instance, the ontological time corresponds to a part of the artwork that has not been created yet, and it decreases as well as the intrinsic time.

Such a conception of creativity has been supported by Sergieu Celibidache who was a Romanian conductor, composer and teacher. His approach to music making is often characterized by what he abstained from rather than what he actively pursued [43]. He was principal conductor of the Munich Philharmonic, Berlin Philharmonic and several European orchestras. Later in life, he taught at the Mainz University in Germany and the Curtis

Institute of Music in Philadelphia. But he frequently refused to release his performances on commercial recordings throughout his lifetime, since in that manner, a listener might not obtain the original observation [44]. Many recordings of his performances were released posthumously. In his insistence on the originality of artistic expression, some discern echoes of Zen Buddhism, exemplified by the concept "ichi-go ichi-e", which translates to "for this time only", "never again", or "one chance in a lifetime". The term reminds one to cherish any gathering that he partakes in, citing the fact that it is original and unique.

Figure 9. Charlotte Caspers replicating her own painting. The artist has spent nearly twice as much time on creating replicas compared to the original [41].

The ontological time that is a part of the artwork yet to be observed or created aligns with the concept of reverse perspective, which concerns a harmonious fusion of composition and construction. The traditional iconography presents a future kingdom of the Lord, which is concurrent to a dyadic interpretation of the creative process [45]. On the other hand, the modern painting presents the psychological time implying the linear order of real numbers. Concepts are confronted in the Presentation of the Virgin, which is a traditional icon that was painted by Tintoretto as well (Figure 10).

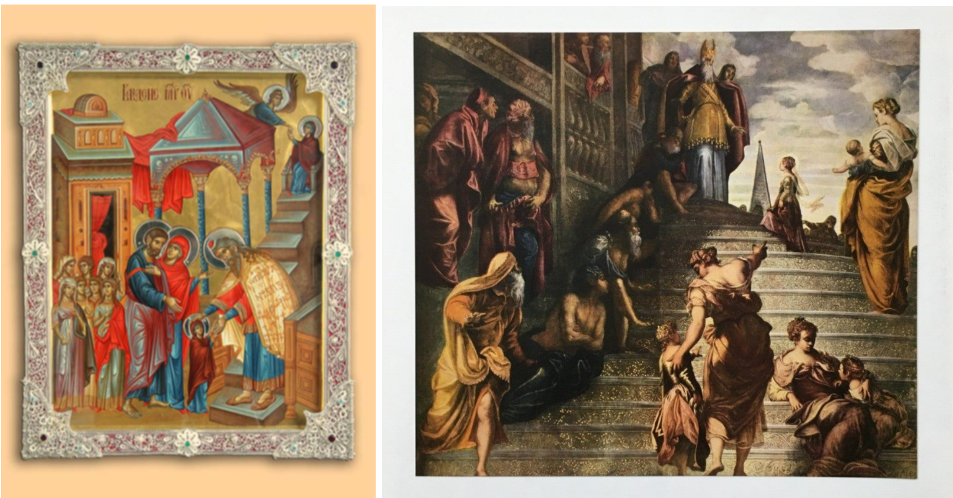

Figure 10. Presentation of the Virgin. (**left**) The traditional icon in the reverse perspective [24]. (**right**) The modern painting in the linear perspective [46].

Psychological time is depicted through the narrowing staircase in the linear perspective, being the overall content of modern painting. On the opposite, the ontological time, which is a supplementary concept, has emerged in the reverse perspective of the icon.

5. Conclusions

In the statistical model of artwork signal processing, two arrows of time emerge which are opposed and dual to each other. The intrinsic time, corresponding to the time operator of wavelets, is established through a multiscale representation of traditional iconography. The psychological time, associated with a spatial domain, is linked to the default timeline of music. The sensation of temporality, manifested through an expansion of the timeline, is presented by the Rényi map. The evolution follows the increase in irreducible randomness, which is akin to the formulation of the second law.

A dual statement, addressing the increase in complexity, defines self-organization. It has established the arrow corresponding to organization and creativity, which corresponds to intrinsic time. The concept is concurrent to an ontological time which refers to the part of an artwork having been unobserved or uncreated. An interrelation between these concepts is illustrated by the spirals of real or dyadic numbers, which completes the paper's aim to elucidate the problem of the time arrow in terms of artistic creation.

The model is intricately linked to the theory of musical forms, where symmetry assumes a central role. It has been corroborated as well by experimental results concerning the originality issue. Additionally, a correlation with the perspective problem is noted. The model has a substantial implication for art theory in terms of vertical time, which is the conception that primarily applies to poetry. Some repercussions to neuroaesthetics and psychophysics have already been indicated. It is eventually related to concepts of observation and measurement, which is the crucial problem in physics requiring a general resolution.

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Appendix A

Wavelets on the unit interval are orthonormal bases ψ_i of the space $L^2(\mathbb{I}) \ominus \mathbb{1}$, whereby indices $i = \frac{2k-1}{2^{j+1}}$ are dyadic fractions between 0 and 1. Their evolution is governed by the axiom $U^\dagger \psi_i = \frac{1}{\sqrt{2}} \psi_{Ri}$ wherein $U^\dagger : f(\cdot) \mapsto \frac{f(\frac{\cdot}{2}) + f(\frac{\cdot+1}{2})}{2}$ is an adjoint of the evolutionary operator $U : f(\cdot) \mapsto f(R\cdot)$. The Rényi map $R : .\tau_1\tau_2 \ldots \mapsto .\tau_2 \ldots$ is a symmetry of the unit interval, corresponding to a unilateral shift in time series of binary digits which concern the measurement process. It has preserved the Lebesgue measure $\mu\, R^{-1} = \mu$ which implies that U is an isometry, i.e., the adjoint is a left inverse $U^\dagger U = I$.

The evolution by U represents an exact system, meaning the limit $\mu(R^t A) \to 1$ for any non-null domain $\mu(A) \neq 0$. Exactness implies mixing, which is an ergodic property of systems which have provided the time operator [47]. Such an operator $T\psi_i = t\psi_i$, that has multiplied the wavelet ψ_i by an eigenvalue $t = \log_2 |i|_2$ corresponding to the dyadic log-norm of its index, is defined on a dense subset of $L^2(\mathbb{I}) \ominus \mathbb{1}$. It follows the commutator relation $[U^\dagger, T] = U^\dagger \Rightarrow [T, U] = U$ which has constituted the time operator of wavelets [31].

An exact system preserving the measure is obviously induced by a non-invertible map. The same holds for U which is not a unitary operator only because there is no inverse. But any exact system naturally extends to the Kolmogorov one which is invertible [47]. The natural extension of R concerns baker's map $B : \ldots\tau_0.\tau_1\tau_2\ldots \mapsto \ldots\tau_0\tau_1.\tau_2\ldots$ which is a bilateral shift preserving the product measure $\mu^2 = \mu \otimes \mu$. The time operator of the Haar wavelet is extended as well as the evolutionary operator, and the commutator relation holds. A paradigmatic wavelet is the Haar one $\psi : .\tau_1\tau_2 \mapsto 1 - 2\tau_1$ producing by translation and normalized dilatation an orthonormal base of $L^2(\mathbb{I}) \ominus \mathbb{1}$.

In terms of the base, extended space should decompose via the identity

$$F(\ldots\tau_0.\tau_1\tau_2\ldots) = A(\ldots\tau_0) \otimes 1 + \sum D_i(\ldots\tau_0) \otimes \psi_i(.\tau_1\tau_2\ldots) \tag{A1}$$

wherein $A(\ldots\tau_0)$ is an approximation and details $\mathbf{D}(\ldots\tau_0.i) = D_i(\ldots\tau_0)$ correspond to the function of dyadic fractions $\ldots\tau_0.i$ presenting a multiresolution analysis [48]. In that regard, the evolutionary operator concerns a doublement of the dyadic argument $U : \mathbf{D}(\cdot) \mapsto \frac{1}{\sqrt{2}}\mathbf{D}(2\cdot)$. The log-norm of an argument relates to the scale of multiresolution which is a concept that has originated from optics and computer vision [49]. Such a dyadic interpretation has temporally decomposed the space into detail subspaces corresponding to wavelets at a certain scale [45].

The evolutionary operator U is the Koopman one, which means that it is evolving random variables over the probability space [47]. In that respect, the Frobenius–Perron operator $U^\dagger$ governs the evolution of distribution densities over the same space. The distribution is represented by a density $\rho = |F|^2$ which is the absolute square of a state evolving in the same manner $U^\dagger\rho = |U^\dagger F|^2$. The main advantage of using operators to present an evolution is the avoidance of point trajectories in order to distinguish a common behavior that is not affected by particular points but continuum-powered domains only.

To emphasize such an insight, one considers the delta distribution δ_τ corresponding to a single point. It should evolve in the manner of $U^\dagger\delta_\tau = \delta_{B\tau}$, that is induced by the baker's map which acts onto particular elements of the domain. The evolution is reversible due to the fact that time does not operate on delta distributions which are pure states similar to orbits in classical mechanics [3]. But irreversibility is implied by the time operator, which indicates a non-local behavior [5]. In that regard, there is a change in representation $\Lambda = \lambda(T)$ transfiguring the reversible evolution into an irreversible one. It applies as well to the delta distribution that is transfigured into an entity having non-null support, which should evolve by the Markov operator $W^\dagger = \Lambda U^\dagger \Lambda^{-1}$. The representation satisfies the properties [33]

- Preservation of equilibrium $1 = \Lambda 1$;
- Preservation of integral $\int \rho = \int \Lambda\rho$;
- Preservation of positivity $\rho \geq 0 \Rightarrow \Lambda\rho \geq 0$;
- Invertibility Λ^{-1} on a dense subset;
- Evolvability $\rho \geq 0 \Rightarrow W^\dagger\rho \geq 0$;
- Irreversibility $\rho \geq 0 \not\Rightarrow W^{-1\dagger}\rho \geq 0$.

Details $\mathbf{D}$ are transfigured into a Markovian tree of hidden variables $\mathbf{S} = \Lambda^{-1\dagger}\mathbf{D}$ which constitutes the wavelet-domain hidden Markov model [34]. Information content is decomposed via the equality $H(\mathbf{D}) = H(\mathbf{S}) + H(\mathbf{D}|\mathbf{S})$ wherein the second term corresponds to an irreducible randomness which persists even after every correlation is subsumed. What matters is the maximization of such a term in order to exhaust the free entropy of details. In that instance, $\mathbf{S}$ is the minimal sufficient statistics representing a causal state of the model [16]. The information $H(\mathbf{S})$ it contains is termed the global complexity, which is a measure of increasing the local complexity $H(S_i)$ over time that relates to the dyadic log-norm of indices [11].

The separation of complexity from irreducible randomness concerns the recognition of an underlying dynamics which is principally neglected by the theory of stochastic processes [50]. One should often assume the domain consisted of particular elements, which is an extensional frame not addressing at all the intensional procedure that has produced it. That is precisely why the measurement problem occurs not only in quantum mechanics only but in classical as well [14]. The measurement process is possible due to an irreversibility represented by the time operator [51].

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