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Research Article

Assessment of Solar Panel Using Multiattribute Decision-Making Approach Based on Intuitionistic Fuzzy Aczel Alsina Heronian Mean Operator

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Multiattribute decision-making (MADM) approach is an effective method for handling ambiguous information in a practical situation. The process of the MADM technique has drawn a lot of interest from various academic and selection processes of extensive analysis. The aggregation operators (AOs) are the best mathematical tools and received a lot of attention from researchers. This article explored the theory of intuitionistic fuzzy IF sets (IFSs) and their certain fundamental operations. The theory of triangular norms also explores Aczel Alsina operations (AAOs) in advanced mathematical tools. The concepts of Heronian mean (HM) and geometric HM (GHM) operators are presented to define interrelationships among different opinions. We developed a list of certain AOs by utilizing AAOs under the system IF information, namely, IF Aczel Alsina HM (IFAAHM), IF Aczel Alsina weighted HM (IFAAWHM), IF Aczel Alsina GHM (IFAAGHM), and IF Aczel Alsina weighted GHM (IFAAWGHM) operators. Some particular characteristics of our invented methodologies are also presented. Solar energy is an effective, efficient resource to enhance electricity production and the country's economic growth. Therefore, we studied an application of solar panel systems to solve real-life problems under a robust technique of the MADM approach by utilizing our invented approaches of IFAAWHM and IFAAWGHM operators. A numerical example was also given to select more suitable solar panels under our proposed methodologies. To find the competitiveness and feasibility of discussed methodologies, we make an inclusive comparative study in which we contrast the results of existing AOs with the consequences of current approaches.

1. Introduction

Decision-making (DM) is an effective method for handling complex and ambiguous information in particle situations. The process of MADM has drawn a lot of interest from various academics, and several of them have recently been the subject of extensive analysis. One practical and natural process used in daily life is the MADM technique [1–3]. Given the system's increasing complexity daily, it is challenging for the decision-maker to select the best alternative from the available ones. As a result, one can comprehend decision-makers' motives for processing such a solid idea before making a choice. Decision-makers find it challenging to give up the idea of optimum since the exact sciences frequently influence human culture. Once the DM process deals with a complicated organizational context making a conclusion based exclusively on one criterion seems insufficient. It is challenging not to consolidate into a single purpose the range of opinion motivations and objectives in companies. Therefore, it is reasonable to suppose that decisions, whether made by a group or an individual, frequently involve multiple competing goals. Thus, trying to accomplish multiple goals at once is more realistic in many circumstances. According to this remark, real-world problems must be resolved optimally by criteria that forbid an ideal solution optimal for each decision-maker under each criterion. As a result, developing a method that facilitates DM has received greater attention in recent years, increasing the systems' life span. It has been widely believed that information relating to alternatives in terms of criteria and associated weights is expressed as clear values. However, it is widely acknowledged that most decisions in real-life circumstances are made in scenarios with often ambiguous or imprecise goals and limitations.

In addition, Zadeh [4] gave the theory of fuzzy set (FS) that includes membership value (MV) which is restricted to the unit interval $[0, 1]$. One of the difficult tasks in an uncertain environment is information aggregation, and different AOs have been devised in various situations. FS is a very effective tool to solve MADM problems, but it is unable to resolve some complicated issues, such as when a person is presented with a yes or no choice of facts. Atanassov [5] investigated the theory of an intuitionistic fuzzy set (IFS), which expands FS to cope with ambiguous information in daily life issues. In IFSs, $\sigma \in [0, 1]$ represents a membership value (MV), and $\upsilon \in [0, 1]$ represents a nonmembership value (NMV). In IFSs, sum of MV and NMV is less than or equal to one, i.e., $0 \leq \sigma + \upsilon \leq 1$. Garg and Rani [6] worked on distance measures and utilized the concepts of different triangular norms to overcome complexities in the aggregation process under the IF environment. Thao and Chou [7] anticipated the concepts of similarity measures and gave some appropriate tools to handle dubious and imprecision information about IFSs. Mahanta and Panda [8] gave the solution to real-life problems by utilizing the idea of several similar measures based on the IF information.

The AOs are an effective and efficient tool for accommodating vague and complex information. Xu [9] introduced some new AOs to evaluate complex and complicated information of human opinions based on FSs. Ye [10] presented arithmetic AOs based on IFSs while Xu and Yager [11] discovered new geometric AOs based on IFSs. Rahman et al. [12] explored the theory of Einstein aggregation models and gave certain aggregation tools based on IFSs. Ullah et al. [13] developed some AOs based on interval-valued T-spherical fuzzy sets. Hayat et al. [14] put forward new AOs based on the IF soft set. Ali et al. [15] gave some new approaches to complex IF soft sets. Garg [16] presented certain aggregation models of IFSs based on Hamacher aggregation tools to a MADM approach. Garg and Rani [17] generalized new AOs based on complex IFSs. Liu and Wang [18] enhanced the concepts of IFSs and improved traditional laws into improved interactional operations to introduce a list of AOs based on HM operators. Kumar and Chen [19] determined a series of new AOs based on improved linguistic IFS to solve MADM approaches. Kaur and Garg [20] enhanced the idea of IFSs in the form of cubic IFSs and gave some particular AOs to solve real-life situations based on MADM approaches. Garg and Kumar [21] investigated possibility degree measures and establish a series of AOs based on linguistic IFSs. Peng et al. [22] introduced a list of certain AOs by considering Hesitant IF information based on Archimedean aggregation models. Akram et al. [23] elaborated the concepts of complex IFSs and provided a series of new AOs based on Hamacher aggregation models.

First, Klement [24] utilized the theory of triangular norm (T-N) and triangular conorm (T-CN) to aggregate fuzzy information in 1982. The T-N is an efficient tool for interpreting complex and complicated fuzzy information. Qualities of the T-N and T-CN have algebraic and analytical properties, including addition and multiplication generators. Hussain et al. [25] elaborated the theory of fuzzy rough set based on appropriate models of Dombi T-Ns and also discovered certain aggregation approaches. Deschrijver and Kerre [26] developed a series of appropriate AOs with the concepts of union, intersection, sum, and productive role using the theory of T-N and T-CN to face uncertainty and inconsistency in the information. Several researchers provided different AOs by using the concepts of T-Ns, and T-CNs such as algebraic sum, Algebraic product, Dombi T-N and T-CN, Hamacher T-N and T-CN, Nilpotent T-N and T-CN, Frank T-N and T-CN, Drastic T-N and T-CN, and Lukasiewicz T-N and T-CN. Xu [27] introduced some appropriate AOs by using the algebraic sum and algebraic product of T-N and T-CN. Ullah et al. [28] developed some AOs using the similarity measures theory based on t-spherical FSs. Mahnaz et al. [29] anticipated a list of certain AOs by using the concepts of Frank T-N and T-CNM based on T-spherical FSs (T-SFSs). Rasuli [30] explored drastic T-N and T-CN to accommodate the uncertainty and inconsistency of fuzzy logic information. Klement and Navara [31] discussed the characteristics of Lukasiewicz T-N and T-CN based on a fuzzy set and its complement fuzzy set.

The theory of Aczel Alsina T-N and T-CN is more flexible than other previously discussed T-N and T-CNs. For the first time, the concept of Aczel Alsina T-N and T-CN under an environment of the classifications of quasilinear functions was given by Aczél and Alsina [32]. Butnariu and Klement [33] elaborated the theory of Aczel Alsina T-N and T-CN under the system of the statistical environment. Farahbod and Eftekhari [34] compared several T-Ns and T-CNs to select and classify more suitable T-N and T-CN. After classification and evaluation of different T-Ns and T-CNs, they observed that Aczel Alsina is more inclusive than other aggregation models. Senapati et al. [35] generated certain aggregation models of IFSs based on power aggregation tools to solve a MADM approach. Senapati and Chen [36] also generalized the concept of IFSs in the interval-valued IFS (IVIFS) framework. They gave an application in which the selection process was deeply described based on the MADM technique. Hussain et al. [37] extended the theory of Pythagorean fuzzy sets (PyFSs) by utilizing the basic operations of Aczel Alsina T-N and T-CN. Hussain et al. [38] elaborated concepts of T-spherical fuzzy sets to select suitable alternatives under the MADM technique by using the operational laws of Aczel Alsina T-N and T-CN.

The HM operator is an efficient tool to aggregate non-negative positive real numbers. Guan and Zhu [39] generalized inequalities and properties of HM operators under FSs. Yu [40] proposed a MADM technique and discovered some new AOs using the HM operator's basic role. Dejian and Yingyu [41] generalized IFSs in the framework of interval-valued IFSs by using the literature on HM operators and presented their application on the MADM technique. Zhang et al. [42] expressed normal intuitionistic fuzzy HM operators using Hamacher T-N and T-CN. Recently, numerous research scholars worked on solar energy systems to facilitate renewable energy resources. Türk et al. [43] provided a multicriteria DM technique to choose suitable solar panels under the system of IFSs. Hu et al. [44] evaluated a suitable solar power system and its installation by using a MADM process based on interval-valued IFSs. Cavallaro et al. [45] utilized the MADM technique to select suitable energy resources for generating electricity based on IF information. Liu et al. [46] worked on linguistic IF value (IFV) by using the HM operator concept. They also proposed a MADM technique to select a suitable research assistant for a public university.

All previously discussed Hamy mean, Bonferroni mean, and Maclaurin symmetric mean operators face some difficulties and cannot handle the loss of information during the aggregation process. Keeping in mind the significance of HM and GHM operators, we studied the theory of HM and GHM tools in the framework of the IF information. We also explored some realistic methods to overcome the loss of information and express how to deal with dubious and uncertain information during the decision-making process. Some robust mathematical tools in the form of Aczel Alsina operations and their deserved properties are also presented. We developed a list of certain methodologies with the help of HM and GHM tools based on AAOs in the framework of the IF information, namely, IFAAHM, IFAAWHM, IFAAGHM, and IFAAWGHM operators. We also explore some appropriate characteristics of our proposed methodologies. To find the practicability and validity of discussed approaches, we applied to select suitable solar panels to enhance energy resources for a public hospital. A numerical example is also established to assess the best solar panel using the IF information. We make an inclusive comparison to compare the results of existing AOs with our proposed approaches.

This manuscript's outline is presented as follows: in Section 2, we study the history of our research work to familiarize the advantages and drawbacks of a fuzzy environment. Section 3 explored specific mathematical tools of Aczel Alsina operations with a numerical example. Section 4 presented a list of new AOs of IFSs based on AAOs, namely, IFAAHM and IFAAWHM operators. Section 5 also established a series of prominent AOs, including IFAAGHM and IFAAWGHM operators. In Section 6, we evaluated an application of a solar panel system under a robust technique of the MADM approach by utilizing our invented methodologies. In Section 7, to verify the reliability and effectiveness of the current discussed techniques, we make an inclusive comparison in which we compare the results of existing methodologies with the results of current AOs. In Section 8, we summarised the whole article.

2. Preliminaries

In the following section, we discuss the basic ideas of IFS, notions of Aczel Alsina, and its basic operations for further development of this article. Moreover, we explore all abbreviations and their meaning in Table 1.

Definition 1 (see [5]). An IFS $\mathfrak{B}$ is the form as follows:

$$\mathfrak{B} = \{(x, (\sigma_{\mathfrak{B}}(x), v_{\mathfrak{B}}(x)))\}, \tag{1}$$

where MV is denoted by $\sigma_{\mathfrak{B}}(x)$, and NMV is denoted by $v_{\mathfrak{B}}(x)$, i.e., $\sigma_{\mathfrak{B}}(x), v_{\mathfrak{B}}(x) \in [0,1]$. The hesitancy value is denoted by $\gamma(x) = 1 - (\sigma_{\mathfrak{B}}(x) + v_{\mathfrak{B}}(x)), \gamma(x) \in [0,1]$, and $\Gamma = (\sigma_{\mathfrak{B}}(x), v_{\mathfrak{B}}(x))$ denotes an IFV such that

$$0 \leq \sigma_{\mathfrak{B}}(x) + v_{\mathfrak{B}}(x) \leq 1. \tag{2}$$

Definition 2 (see [32]). The notion of Aczel Alsina TN is in the following form:

$$Ŧ_{Á}^{\mathbb{G}}(a, ḅ) = \left\{ \begin{array}{ll} Ŧ_D(a, ḅ) \text{ if } \mathbb{G} = 0 & \\ \min(a, ḅ) \text{ if } \mathbb{G} = \infty & \forall, 0 \leq \mathbb{G} \leq +\infty \\ e^{-\left((-\log a)^{\mathbb{G}} + (-\log ḅ)^{\mathbb{G}}\right)^{1/\mathbb{G}}} \text{ otherwise} & \end{array} \right\}, \tag{3}$$

and the notion of Aczel Alsina T-CN in the following form:

$$\dot{Ś}_{Á}^{\mathbb{G}}(a, ḅ) = \left\{ \begin{array}{ll} \dot{Ś}_D(a, ḅ) \text{ if } \mathbb{G} = 0 & \\ \max(a, ḅ) \text{ if } \mathbb{G} = \infty & \forall, 0 \leq \mathbb{G} \leq +\infty \\ 1 - e^{-\left((-\log a)^{\mathbb{G}} + (-\log ḅ)^{\mathbb{G}}\right)^{1/\mathbb{G}}} \text{ otherwise} & \end{array} \right\}, \tag{4}$$

respectively.

Definition 3 (see [47]). Let $\Gamma = (\sigma_\Gamma(x), v_\Gamma(x))$ be an IFV, then, the score function $\hat{S}(\Gamma)$ is defined as follows:

$$\hat{S}(\Gamma) = \sigma_\Gamma(x) - v_\Gamma(x) \in [-1, 1]. \tag{5}$$

Definition 4 (see [48]). Let $\Gamma = (\sigma_\Gamma(x), v_\Gamma(x))$ be an IFV, then, the accuracy function $Æ(\Gamma)$ is defined as follows:

$$Æ(\Gamma) = \sigma_\Gamma(x) + v_\Gamma(x) \in [0, 1]. \tag{6}$$

Remark 1 (see [48]). If $\Gamma = (\sigma_\Gamma(x), v_\Gamma(x))$ and $\mathcal{T} = (\sigma_{\mathcal{T}}(x), v_{\mathcal{T}}(x))$ are two IFVs. Then,

(i) $\hat{S}(\Gamma) < \hat{S}(\Gamma)$, if $\Gamma < \mathcal{T}$

(ii) $\hat{S}(\Gamma) > \hat{S}(\Gamma)$, if $\Gamma > \mathcal{T}$

(iii) $\hat{S}(\Gamma) = \hat{S}(\Gamma)$ then:

(a) $Æ(\Gamma) > Æ(\Gamma)$, if $\Gamma > \mathcal{T}$

(b) $Æ(\Gamma) < Æ(\Gamma)$, if $\Gamma < \mathcal{T}$

(c) $Æ(\Gamma) = Æ(\Gamma)$, if $\Gamma \approx \mathcal{T}$

Table 1: Abbreviations and their meanings.

Abbreviation	Meanings
MADM	Multiattribute decision-making
AOs	Aggregation operators
IFSs	Intuitionistic fuzzy sets
HM	Heronian mean
GHM	Geometric Heronian mean
AAOs	Aczel Alsina operations
MV	Membership value
NMV	Nonmembership value
IFAAHM	Intuitionistic fuzzy Aczel Alsina Heronian mean
IFAAWHM	Intuitionistic fuzzy Aczel Alsina weighted Heronian mean
IFAAGHM	Intuitionistic fuzzy Aczel Alsina geometric Heronian mean
IFAAWGHM	Intuitionistic fuzzy Aczel Alsina weighted geometric Heronian mean
IVIFDHM	Interval-valued intuitionistic fuzzy Dombi Hamy mean
T-N	t-norm
T-CN	t-conorm
IFVs	Intuitionistic fuzzy values
DM	Decision-making
IFGWHM	Intuitionistic fuzzy geometric weighted Heronian mean
IFWA	Intuitionistic fuzzy weighted average
IFWG	Intuitionistic fuzzy weighted geometric
IFDWA	Intuitionistic fuzzy Dombi weighted average
IFDWG	Intuitionistic fuzzy Dombi weighted geometric
IFEWA	Intuitionistic fuzzy Einstein weighted average
IFEWG	Intuitionistic fuzzy Einstein weighted geometric
IVIFHM	Interval-valued intuitionistic fuzzy Hamy mean
IVIFWA	Interval-valued intuitionistic fuzzy weighted average

Example 1. Let $\Gamma_1 = (0.16, 0.35), \Gamma_2 = (0.56, 0.28), \Gamma_3 = (0.62, 0.25)$ be three IFVs. Then, the score values and accuracy values of the given IFVs are as follows:

$$\begin{aligned}\hat{S}(\Gamma_1) &= 0.16 - 0.35 \\ &= -0.19 \in [-1, 1], \\ \hat{S}(\Gamma_2) &= 0.56 - 0.28 \\ &= 0.28 \in [-1, 1], \\ \hat{S}(\Gamma_3) &= 0.62 - 0.25 \\ &= 0.37 \in [-1, 1].\end{aligned} \tag{7}$$

Now, we find the value of the accuracy function as follows:

$$\begin{aligned}Æ(\Gamma_1) &= 0.16 + 0.35 \\ &= 0.51 \in [0, 1], \\ Æ(\Gamma_2) &= 0.56 + 0.28 \\ &= 0.84 \in [0, 1], \\ Æ(\Gamma_3) &= 0.62 + 0.25 \\ &= 0.87 \in [0, 1].\end{aligned} \tag{8}$$

We recall the theory of the HM operator and its basic properties. Moreover, we also study GHM operators with their basic characteristics.

Definition 5 (see [40]). Consider $\Gamma_i = (\sigma_{\Gamma_i} \upsilon_{\Gamma_i})$ be the family of IFVs with $p > 0, q > 0$, then, the HM operator is defined as follows:

$$\mathrm{HM}(\Gamma_1, \Gamma_2, \ldots, \Gamma_и) = \left(\frac{2}{и(и+1)} \sum_{i=1}^{и} \sum_{j=1}^{и} \sqrt{\Gamma_i^p . \Gamma_j^q}\right)^{1/p+q}. \tag{9}$$

HM must satisfy the following conditions:

(1) If all $a_i = 0, \forall i$, then, the $\mathrm{HM}^{p,q}(0, 0, \ldots, 0) = 0$
(2) If all $a_i = a, \forall i$, then, the $\mathrm{HM}^{p,q}(a_1, a_2, \ldots, a_и) = a$
(3) If all $a_i \geq b_i, \forall i$, then, the $\mathrm{HM}^{p,q}(a_1, a_2, \ldots, a_и) \geq \mathrm{HM}^{p,q}(b_1, b_2, \ldots, b_и)$
(4) $\min\{\Gamma_i\} \leq \mathrm{HM}^{p,q}(\Gamma_1, \Gamma_2, \ldots, \Gamma_и) \leq \max\{\Gamma_i\}$.

Definition 6. (see [40]). Consider $\Gamma_i = (\sigma_{\Gamma_i} \upsilon_{\Gamma_i})$ to be the family of IFVs and $p > 0, q > 0$. Then, the GHM operator is defined as follows:

$$\mathrm{GHM}^{p,q}(\Gamma_1, \Gamma_2, \ldots, \Gamma_и) = \left(\frac{1}{p+q} \prod_{i=1}^{и} \prod_{j=1}^{и} p\Gamma_i + q\Gamma_j\right)^{2/и(и+1)}. \tag{10}$$

GHM must satisfy the following condition:

(1) If all $\Gamma_i = 0, \forall i$, then, the $\mathrm{GHM}^{p,q}(0, 0, \ldots, 0) = 0$
(2) If all $\Gamma_i = \Gamma, \forall i$, then, the $\mathrm{GHM}^{p,q}(\Gamma_1, \Gamma_2, \ldots, \Gamma_и) = \Gamma$

(3) If all $a_i \geq b_i, \forall i$, then, the $\text{GHM}^{p,q}(a_1, a_2, \ldots, a_{\text{и}}) \geq \text{GHM}^{p,q}(b_1, b_2, \ldots, b_{\text{и}})$

(4) $\min\{a_i\} \leq \text{GHM}^{p,q}(a_1, a_2, \ldots, a_{\text{и}}) \leq \max\{a_i\}$.

3. Aczel Alsina Operations

We explore Aczel Alsina operations in the form of algebraic sum, product, scalar multiplication, and power role under the IF information system.

Definition 7 (see [48]). Let $\Gamma = (\sigma(x), v(x))$, $\Gamma_1 = (\sigma_1(x), v_1(x))$, and $\Gamma_2 = (\sigma_2(x), v_2(x))$ be three IFVs, and $\lambda > 0, \mathfrak{G} \geq 1$ be any real numbers. Then, some basic operations of IFVs based on Definition 2, are given as follows:

(1) $\Gamma_1 \oplus \Gamma_2 = \left(1 - e^{-((-\ln(1-\sigma_1(x)))^{\mathfrak{G}} + (-\ln(1-\sigma_2(x)))^{\mathfrak{G}})^{1/\mathfrak{G}}}, e^{-((-\ln(v_1(x)))^{\mathfrak{G}} + (-\ln(v_2(x)))^{\mathfrak{G}})^{1/\mathfrak{G}}}\right)$

(2) $\Gamma_1 \otimes \Gamma_2 = \left(e^{-((-\ln(\sigma_1(x)))^{\mathfrak{G}} + (-\ln(\sigma_2(x)))^{\mathfrak{G}})^{1/\mathfrak{G}}}, 1 - e^{-((-\ln(1-v_1(x)))^{\mathfrak{G}} + (-\ln(1-v_2(x)))^{\mathfrak{G}})^{1/\mathfrak{G}}}\right)$

(3) $\lambda\Gamma = \left(1 - e^{-(\lambda(-\ln(1-\sigma(x)))^{\mathfrak{G}})^{1/\mathfrak{G}}}, e^{-(\lambda(-\ln(v(x)))^{\mathfrak{G}})^{1/\mathfrak{G}}}\right)$

(4) $\Gamma^{\lambda} = \left(e^{-(\lambda(-\ln(\sigma(x)))^{\mathfrak{G}})^{1/\mathfrak{G}}}, 1 - e^{-(\lambda(-\ln(1-v(x)))^{\mathfrak{G}})^{1/\mathfrak{G}}}\right)$.

4. Intuitionistic Fuzzy Aczel Alsina Weighted Heronian Mean Operators

In this part, we established some new operators using the operational laws of Aczel Alsina. To find the flexibility of AOs, we also present some characteristics of our proposed approaches.

Definition 8. Consider $\Gamma_i = (\sigma_{\Gamma_i} v_{\Gamma_i})$ be the family of IFV and $p > 0, q > 0$. Then, an IFAAHM is defined as follows:

$$\text{IFAAHM}^{p,q}(\Gamma_1, \Gamma_2, \Gamma_3, \ldots, \Gamma_{\text{и}}) = \left(\frac{2}{\text{и}(\text{и}+1)} \sum_{i=1}^{\text{и}} \sum_{j=1}^{\text{и}} \left(\Gamma_i^p \times \Gamma_j^q\right)\right)^{1/p+q}. \tag{11}$$

Theorem 1. *Consider $\Gamma_i = (\sigma_{\Gamma_i} v_{\Gamma_i})$ be the family of IFVs and $p > 0, q > 0$, then, the aggregated value of IFAAHM is defined as follows:*

$$\text{IFAAHM}^{p,q}(\Gamma_1, \Gamma_2, \Gamma_3, \ldots, \Gamma_{\text{и}}) = \left(\left(\frac{2}{\text{и}(\text{и}+1)}\right)\left(\underset{i=1}{\overset{\text{и}}{\oplus}} \underset{j=1}{\overset{\text{и}}{\oplus}} \left(\Gamma_i^p \otimes \Gamma_j^q\right)\right)\right)^{1/p+q}$$

$$= \left(\begin{array}{c} e^{-\left((1/p+q)\left(-\ln\left(1-e^{-\left((2/\text{и}(\text{и}+1))\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p(-\ln(\sigma_i(x)))^{\mathfrak{G}}\right)+\left(q\left(-\ln\left(\sigma_j(x)\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}} \\ 1 - e^{-\left((1/p+q)\left(-\ln\left(1-e^{-\left((2/\text{и}(\text{и}+1))\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p(-\ln(1-v_i(x)))^{\mathfrak{G}}\right)+\left(q\left(-\ln\left(1-v_j(x)\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}} \end{array} \right). \tag{12}$$

Proof. Consider $\Gamma_i = (\sigma_{\Gamma_i} v_{\Gamma_i}), i = 1, 2, 3, \ldots, \text{и}$ be the family of IFVs and $p > 0, q > 0$, we have

$$\Gamma_i^p = \left(e^{-\left(p\left(-\ln\left(\sigma_i(x)\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}, 1 - e^{-\left(p\left(-\ln\left(1-v_i(x)\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}} \right),$$

$$\Gamma_j^q = \left(e^{-\left(q\left(-\ln\left(\sigma_j(x)\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}, 1 - e^{-\left(q\left(-\ln\left(1-v_j(x)\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}} \right),$$

$$\left(\Gamma_i^p \otimes \Gamma_j^q\right) = \begin{pmatrix} e^{-\left(\left(p\left(-\ln\left(\sigma_i(x)\right)\right)^{\mathfrak{G}}\right) + \left(q\left(-\ln\left(\sigma_j(x)\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}, \\ 1 - e^{-\left(\left(p\left(-\ln\left(1-v_i(x)\right)\right)^{\mathfrak{G}}\right) + \left(q\left(-\ln\left(1-v_j(x)\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}} \end{pmatrix},$$

$$\oplus_{j=1}^{\text{и}}\left(\Gamma_i^p \otimes \Gamma_j^q\right) = \begin{pmatrix} 1 - e^{-\left(\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(\sigma_i(x)\right)\right)^{\mathfrak{G}}\right)+\left(q\left(-\ln\left(\sigma_j(x)\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}, \\ e^{-\left(\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(1-v_i(x)\right)\right)^{\mathfrak{G}}\right)+\left(q\left(-\ln\left(1-v_j(x)\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}} \end{pmatrix},$$

$$\oplus_{j=1}^{\text{и}}\left(\Gamma_i^p \otimes \Gamma_j^q\right) = \begin{pmatrix} 1 - e^{-\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(\sigma_i(x)\right)\right)^{\mathfrak{G}}\right)+\left(q\left(-\ln\left(\sigma_j(x)\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}, \\ e^{-\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(1-v_i(x)\right)\right)^{\mathfrak{G}}\right)+\left(q\left(-\ln\left(1-v_j(x)\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}} \end{pmatrix},$$

$$\left(\frac{2}{\text{и}(\text{и}+1)}\right)\left(\underset{i=1}{\overset{\text{и}}{\oplus}}\underset{j=1}{\overset{\text{и}}{\oplus}}\left(\Gamma_i^p \otimes \Gamma_j^q\right)\right) = \begin{pmatrix} 1 - e^{-\left((2/\text{и}(\text{и}+1))\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(\sigma_i(x)\right)\right)^{\mathfrak{G}}\right)+\left(q\left(-\ln\left(\sigma_j(x)\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}, \\ e^{-\left((2/\text{и}(\text{и}+1))\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(1-v_i(x)\right)\right)^{\mathfrak{G}}\right)+\left(q\left(-\ln\left(1-v_j(x)\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}} \end{pmatrix},$$

$$\left(\left(\frac{2}{\text{и}(\text{и}+1)}\right)\left(\underset{i=1}{\overset{\text{и}}{\oplus}}\underset{j=1}{\overset{\text{и}}{\oplus}}\left(\Gamma_i^p \otimes \Gamma_j^q\right)\right)\right)^{1/p+q} = \begin{pmatrix} e^{-\left((1/p+q)\left(-\ln\left(1-e^{-\left((2/\text{и}(\text{и}+1))\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(\sigma_i(x)\right)\right)^{\mathfrak{G}}\right)+\left(q\left(-\ln\left(\sigma_j(x)\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}} \\ 1 - e^{-\left((1/p+q)\left(-\ln\left(1-e^{-\left((2/\text{и}(\text{и}+1))\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(1-v_i(x)\right)\right)^{\mathfrak{G}}\right)+\left(q\left(-\ln\left(1-v_j(x)\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}} \end{pmatrix}. \tag{13}$$

□

Example 2. Consider $\Gamma_1 = (0.27, 0.47), \Gamma_2 = (0.72, 0.14), \Gamma_3 = (0.32, 0.54)$ be three IFVs and $p = 2, q = 1, \mathfrak{G} = 3$. Then, the aggregated value of IFAAHM is also an IFV. So we have

$$\text{IFAAHM}^{p,q}(\Gamma_1,\Gamma_2,\Gamma_3,\ldots,\Gamma_\text{и}) = \left(e^{-\left((1/p+q)\left(-\ln\left(1-e^{-\left((2/\text{и}(\text{и}+1))\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(\sigma_i(x)\right)\right)^{\mathfrak{G}}\right)+\left(q\left(-\ln\left(\sigma_j(x)\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}, 1-e^{-\left((1/p+q)\left(-\ln\left(1-e^{-\left((2/\text{и}(\text{и}+1))\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(1-v_i(x)\right)\right)^{\mathfrak{G}}\right)+\left(q\left(-\ln\left(1-v_j(x)\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}} \right)$$

$$= \left(e^{-\left((1/3)\left(-\ln\left(1-e^{-\left((1/6)\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\left(-\ln\left(1-e^{-\left(\left(2\left(-\ln\left(\sigma_i(x)\right)\right)^{3}\right)+\left(1\left(-\ln\left(\sigma_j(x)\right)\right)^{3}\right)\right)^{1/3}}\right)\right)^{3}\right)\right)^{1/3}}\right)\right)^{3}\right)^{1/3}}, 1-e^{-\left((1/3)\left(-\ln\left(1-e^{-\left((1/6)\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\left(-\ln\left(1-e^{-\left(\left(2\left(-\ln\left(1-v_i(x)\right)\right)^{3}\right)+\left(1\left(-\ln\left(1-v_j(x)\right)\right)^{3}\right)\right)^{1/3}}\right)\right)^{3}\right)\right)^{1/3}}\right)\right)^{3}\right)^{1/3}} \right)$$

$$= (0.5518, 0.2887). \tag{14}$$

To check the validity of our discussed technique, we explore the basic characteristics of the IFAAHM operators such as idempotency, monotonicity, and boundedness.

Theorem 2 (Idempotency). *Consider $\Gamma_i = (v_{\Gamma_i}(x), v_{\Gamma_i}(x))$, $i = 1,2,3,\ldots,\text{и}$ to be the collection of identical IFVs. Then, $\text{IFAAHM}^{p,q}(\Gamma_1,\Gamma_2,\Gamma_3,\ldots,\Gamma_\text{и}) = \Gamma$. The proof is straight forward.*

Theorem 3 (Monotonicity). *Consider $\Gamma_i = (\sigma_{\Gamma_i}(x), v_{\Gamma_i}(x))$ and $\varsigma_i = (\sigma_{\varsigma_i}(x), v_{\varsigma_i}(x)), i = 1,2,3,\ldots,\text{и}$ to be any two IFVs, if $\Gamma_i \leq \varsigma_i, \forall, i$. Then,*

$$\text{IFAAHM}^{p,q}(\Gamma_1,\Gamma_2,\Gamma_3,\ldots,\Gamma_\text{и}) \leq \text{IFAAHM}^{p,q}(\varsigma_1,\varsigma_2,\varsigma_3,\ldots,\varsigma_\text{и}). \tag{15}$$

Proof. Consider $\Gamma_i = (\sigma_{\Gamma_i}(x), v_{\Gamma_i}(x)), i = 1,2,3,\ldots,\text{и}$ be the family of IFVs and $\Gamma_i \leq \varsigma_i, \forall, i$. Then, we have to show that $\text{IFAAHM}^{p,}\ q(\Gamma_1,\Gamma_2,\Gamma_3,\ldots,\Gamma_\text{и}) \leq \text{IFAAHM}^{p,q}\ (\varsigma_1,\varsigma_2,\varsigma_3,\ldots,\varsigma_\text{и})$.

$$= \left(e^{-\left((1/p+q)\left(-\ln\left(1-e^{-\left((2/\text{и}(\text{и}+1))\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(\sigma_i(x)\right)\right)^{\mathfrak{G}}\right)+\left(q\left(-\ln\left(\sigma_j(x)\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}, 1-e^{-\left((1/p+q)\left(-\ln\left(1-e^{-\left((2/\text{и}(\text{и}+1))\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(1-v_i(x)\right)\right)^{\mathfrak{G}}\right)+\left(q\left(-\ln\left(1-v_j(x)\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}} \right)$$

$$\leq \left(e^{-\left((1/p+q)\left(-\ln\left(1-e^{-\left((2/\text{и}(\text{и}+1))\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(\sigma_{\varsigma_i}\right)\right)^{\mathbb{G}}\right)+\left(q\left(-\ln\left(\sigma_{\varsigma_j}\right)\right)^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}}\right)\right)\right)\right)^{\mathbb{G}}}\right)\right)^{1/\mathbb{G}}\right)^{\mathbb{G}}\right)^{1/\mathbb{G}}}, 1-e^{-\left((1/p+q)\left(-\ln\left(1-e^{-\left((2/\text{и}(\text{и}+1))\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(1-v_{\varsigma_i}\right)\right)^{\mathbb{G}}\right)+\left(q\left(-\ln\left(1-v_{\varsigma_j}\right)\right)^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}}\right)\right)\right)\right)^{\mathbb{G}}}\right)\right)^{1/\mathbb{G}}\right)^{\mathbb{G}}\right)^{1/\mathbb{G}}} \right)$$

$$\text{IFAAHM}^{p,q}\left(\Gamma_1, \Gamma_2, \Gamma_3, \ldots, \Gamma_{\text{и}}\right) \leq \text{IFAAHM}^{p,q}\left(\varsigma_1, \varsigma_2, \varsigma_3, \ldots, \varsigma_{\text{и}}\right). \tag{16}$$

Hence proved. □

Theorem 4 (Boundedness). *Consider* $\Gamma_i = (\sigma_{\Gamma_i}(x), v_{\Gamma_i}(x)), i = 1, 2, 3, \ldots, \text{и}$ *be the family of IFVs, and* $\Gamma_i^- = \min(\Gamma_1, \Gamma_2, \Gamma_3, \ldots, \Gamma_{\text{и}})$ *and* $\Gamma_i^+ = \max(\Gamma_1, \Gamma_2, \Gamma_3, \ldots, \Gamma_{\text{и}})$*, then,*

$$\Gamma_i^- \leq \text{IFAAHM}^{p,q}\left(\Gamma_1, \Gamma_2, \Gamma_3, \ldots, \Gamma_{\text{и}}\right) \leq \Gamma_i^+. \tag{17}$$

Proof. Suppose that $\Gamma_i = (\sigma_{\Gamma_i}(x), v_{\Gamma_i}(x)), i = 1, 2, 3, \ldots, \text{и}$ be the family of IFVs, and $\Gamma_i^- = \min(\Gamma_1, \Gamma_2, \Gamma_3, \ldots, \Gamma_{\text{и}})$, and $\Gamma_i^+ = \max(\Gamma_1, \Gamma_2, \Gamma_3, \ldots, \Gamma_{\text{и}})$, since $\sigma_\Gamma^- = \min(\sigma_{\Gamma_i}(x)), v_\Gamma^+ = \max(v_{\Gamma_i}(x))$ and $\sigma_\Gamma^+ = \max(\sigma_{\Gamma_i}(x)), v_\Gamma^- = \min(v_{\Gamma_i}(x)), \forall, i$. To show $\Gamma_i^- \leq \text{IFAAHM}^{p,q}(\Gamma_1, \Gamma_2, \Gamma_3, \ldots, \Gamma_{\text{и}}) \leq \Gamma_i^+$, we must satisfy the following inequalities:

$$\begin{aligned}
&\left(e^{-\left((1/p+q)\left(-\ln\left(1-e^{-\left((2/\text{и}(\text{и}+1))\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p(-\ln(\sigma^-(x)))^{\mathbb{G}}\right)+\left(q(-\ln(\sigma^-(x)))^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}}\right)\right)^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}}\right)\right)\right)^{\mathbb{G}}\right)^{1/\mathbb{G}}} \right.\\
&\leq e^{-\left((1/p+q)\left(-\ln\left(1-e^{-\left((2/\text{и}(\text{и}+1))\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(\sigma_i(x)\right)\right)^{\mathbb{G}}\right)+\left(q\left(-\ln\left(\sigma_j(x)\right)\right)^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}}\right)\right)^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}}\right)\right)\right)^{\mathbb{G}}\right)^{1/\mathbb{G}}}\\
&\left.\leq e^{-\left((1/p+q)\left(-\ln\left(1-e^{-\left((2/\text{и}(\text{и}+1))\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p(-\ln(\sigma^+(x)))^{\mathbb{G}}\right)+\left(q(-\ln(\sigma^+(x)))^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}}\right)\right)^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}}\right)\right)\right)^{\mathbb{G}}\right)^{1/\mathbb{G}}} \right)\\
&\left(\leq 1-e^{-\left((1/p+q)\left(-\ln\left(1-e^{-\left((2/\text{и}(\text{и}+1))\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p(-\ln(1-v^-(x)))^{\mathbb{G}}\right)+\left(q(-\ln(1-v^-(x)))^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}}\right)\right)^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}}\right)\right)\right)^{\mathbb{G}}\right)^{1/\mathbb{G}}} \right.\\
&\leq 1-e^{-\left((1/p+q)\left(-\ln\left(1-e^{-\left((2/\text{и}(\text{и}+1))\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(1-v_i(x)\right)\right)^{\mathbb{G}}\right)+\left(q\left(-\ln\left(1-v_j(x)\right)\right)^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}}\right)\right)^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}}\right)\right)\right)^{\mathbb{G}}\right)^{1/\mathbb{G}}}\\
&\left. 1-e^{-\left((1/p+q)\left(-\ln\left(1-e^{-\left((2/\text{и}(\text{и}+1))\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p(-\ln(1-v^+(x)))^{\mathbb{G}}\right)+\left(q(-\ln(1-v^+(x)))^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}}\right)\right)^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}}\right)\right)\right)^{\mathbb{G}}\right)^{1/\mathbb{G}}} \right)
\end{aligned} \tag{18}$$

$$\Gamma_i^- \leq \text{IFAAHM}^{p,q}\left(\Gamma_1, \Gamma_2, \Gamma_3, \ldots, \Gamma_{\text{и}}\right) \leq \Gamma_i^+.$$

Hence proved. □

Definition 9. Consider $\Gamma_i = (\sigma_{\Gamma_i}, \upsilon_{\Gamma_i})$ be the family of IFVs with weight vector $\varpi = (\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_{\text{и}})^T$, $\varpi_{\text{и}} \in [0,1]$, $\sum_{i=1}^{\text{и}} \varpi_i = 1$ and $p > 0, q > 0$. Then, an IFAAWHM is defined as follows:

$$\text{IFAAWHM}^{p,q}(\Gamma_1, \Gamma_2, \Gamma_3, \ldots, \Gamma_n) = \left(\text{Б} \sum_{i=1}^{\text{и}} \sum_{j=1}^{\text{и}} \left((\varpi_i \Gamma_i)^p \dot{\otimes} (\varpi_j \Gamma_j)^q \right) \right)^{\wp}, \tag{19}$$

where $\wp = 1/p + q$ and $\text{Б} = 2/\text{и}(\text{и}+1)$.

Theorem 5. *Consider $\Gamma_i = (\sigma_{\Gamma_i}, \upsilon_{\Gamma_i})$ be the family of IFVs with weight vector $\varpi = (\varpi_1, \varpi_2, \varpi_3, \ldots, \varpi_{\text{и}})^T$, $\varpi_{\text{и}} \in [0,1]$, $\sum_{i=1}^{\text{и}} \varpi_i = 1$ and $p > 0, q > 0$. Then, an IFAAWHM is defined as follows:*

$$\text{IFAAWHM}^{p,q}(\Gamma_1, \Gamma_2, \Gamma_3, \ldots, \Gamma_{\text{и}}) = \begin{pmatrix} e^{-\left(\left(\wp\left(-\ln\left(1-e^{-\left(\text{Б}\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-(\alpha_i+\alpha_j)^{1/₲}}\right)\right)^{₲}\right)\right)^{1/₲}}\right)\right)\right)^{₲}\right)^{1/₲}} \\ 1-e^{-\left(\left(\wp\left(-\ln\left(1-e^{-\left(\text{Б}\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-(\beta_i+\beta_j)^{1/₲}}\right)\right)^{₲}\right)\right)^{1/₲}}\right)\right)\right)^{₲}\right)^{1/₲}} \end{pmatrix}, \tag{20}$$

where $\alpha_i = \left(p\left(-\ln\left(1-e^{-(\varpi_i(-\ln(1-\sigma_i(x)))^{₲})^{1/₲}}\right)\right)^{₲}\right)$, $\alpha_j = \left(q\left(-\ln\left(1-e^{-(\varpi_j(-\ln(1-\sigma_j(x)))^{₲})^{1/₲}}\right)\right)^{₲}\right)$ *and* $\beta_i = \left(p\left(-\ln\left(1-e^{-(\varpi_i(-\ln(\upsilon_i(x)))^{₲})^{1/₲}}\right)\right)^{₲}\right)$, $\beta_j = \left(q\left(-\ln\left(1-e^{-(\varpi_j(-\ln(\upsilon_j(x)))^{₲})^{1/₲}}\right)\right)^{₲}\right)$

Proof. Consider the $\Gamma_i = (\sigma_{\Gamma_i}, \upsilon_{\Gamma_i}), i = 1,2,3,\ldots,\text{и}$, we have

$$\varpi_i \Gamma_i = \left(1-e^{-\left(\varpi_i\left(-\ln\left(1-\sigma_i(x)\right)\right)^{₲}\right)^{1/₲}}, e^{-\left(\varpi_i\left(-\ln\left(\upsilon_i(x)\right)\right)^{₲}\right)^{1/₲}}\right),$$

$$\varpi_j \Gamma_j = \left(1-e^{-\left(\varpi_j\left(-\ln\left(1-\sigma_j(x)\right)\right)^{₲}\right)^{1/₲}}, e^{-\left(\varpi_j\left(-\ln\left(\upsilon_j(x)\right)\right)^{₲}\right)^{1/₲}}\right),$$

$$(\varpi_i \Gamma_i)^p = \left(e^{-\left(p\left(-\ln\left(1-e^{-(\varpi_i(-\ln(1-\sigma_i(x)))^{₲})^{1/₲}}\right)\right)^{₲}\right)^{1/₲}}, 1-e^{-\left(p\left(-\ln\left(1-e^{-(\varpi_i(-\ln(\upsilon_i(x)))^{₲})^{1/₲}}\right)\right)^{₲}\right)^{1/₲}}\right),$$

$$(\varpi_j \Gamma_j)^q = \left(e^{-\left(q\left(-\ln\left(1-e^{-(\varpi_j(-\ln(1-\sigma_j(x)))^{₲})^{1/₲}}\right)\right)^{₲}\right)^{1/₲}}, 1-e^{-\left(q\left(-\ln\left(1-e^{-(\varpi_j(-\ln(\upsilon_j(x)))^{₲})^{1/₲}}\right)\right)^{₲}\right)^{1/₲}}\right),$$

$$(\varpi_i \Gamma_i)^p \dot{\otimes} (\varpi_j \Gamma_j)^q = \begin{pmatrix} e^{-\left(\left(p\left(-\ln\left(1-e^{-(\varpi_i(-\ln(1-\sigma_i(x)))^{₲})^{1/₲}}\right)\right)^{₲}\right)+\left(q\left(-\ln\left(1-e^{-(\varpi_j(-\ln(1-\sigma_j(x)))^{₲})^{1/₲}}\right)\right)^{₲}\right)\right)^{1/₲}}, \\ 1-e^{-\left(\left(p\left(-\ln\left(1-e^{-(\varpi_i(-\ln(\upsilon_i(x)))^{₲})^{1/₲}}\right)\right)^{₲}\right)+\left(q\left(-\ln\left(1-e^{-(\varpi_j(-\ln(\upsilon_j(x)))^{₲})^{1/₲}}\right)\right)^{₲}\right)\right)^{1/₲}} \end{pmatrix},$$

$$\sum_{j=1}^{\text{и}}\left((\omega_i\Gamma_i)^p \dot{\otimes} (\omega_j\Gamma_j)^q\right) = \left(\begin{array}{c} 1-e^{-\left(\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(1-e^{-\left(\omega_i\left(-\ln\left(1-\sigma_i(x)\right)\right)^{\text{₵}}\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)+\left(q\left(-\ln\left(1-e^{-\left(\omega_j\left(-\ln\left(1-\sigma_j(x)\right)\right)^{\text{₵}}\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)^{1/\text{₵}}}, \\ e^{-\left(\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(1-e^{-\left(\omega_i\left(-\ln\left(v_i(x)\right)\right)^{\text{₵}}\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)+\left(q\left(-\ln\left(1-e^{-\left(\omega_j\left(-\ln\left(v_j(x)\right)\right)^{\text{₵}}\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)^{1/\text{₵}}} \end{array}\right),$$

$$\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left((\omega_i\Gamma_i)^p \dot{\otimes} (\omega_j\Gamma_j)^q\right) = \left(\begin{array}{c} 1-e^{-\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(1-e^{-\left(\omega_i\left(-\ln\left(1-\sigma_i(x)\right)\right)^{\text{₵}}\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)+\left(q\left(-\ln\left(1-e^{-\left(\omega_j\left(-\ln\left(1-\sigma_j(x)\right)\right)^{\text{₵}}\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)^{1/\text{₵}}}, \\ e^{-\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(1-e^{-\left(\omega_i\left(-\ln\left(v_i(x)\right)\right)^{\text{₵}}\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)+\left(q\left(-\ln\left(1-e^{-\left(\omega_j\left(-\ln\left(v_j(x)\right)\right)^{\text{₵}}\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)^{1/\text{₵}}} \end{array}\right),$$

$$\text{Б}\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left((\omega_i\Gamma_i)^p \dot{\otimes} (\omega_j\Gamma_j)^q\right) = \left(\begin{array}{c} 1-e^{-\left(\text{Б}\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(1-e^{-\left(\omega_i\left(-\ln\left(1-\sigma_i(x)\right)\right)^{\text{₵}}\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)+\left(q\left(-\ln\left(1-e^{-\left(\omega_j\left(-\ln\left(1-\sigma_j(x)\right)\right)^{\text{₵}}\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)\right)^{1/\text{₵}}}, \\ e^{-\left(\text{Б}\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(1-e^{-\left(\omega_i\left(-\ln\left(v_i(x)\right)\right)^{\text{₵}}\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)+\left(q\left(-\ln\left(1-e^{-\left(\omega_j\left(-\ln\left(v_j(x)\right)\right)^{\text{₵}}\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)\right)^{1/\text{₵}}} \end{array}\right),$$

$$\left(\text{Б}\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left((\omega_i\Gamma_i)^p \dot{\otimes} (\omega_j\Gamma_j)^q\right)\right)^{\wp} = \left(\begin{array}{c} e^{-\left(\wp\left(-\ln\left(1-e^{-\left(\text{Б}\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(1-e^{-\left(\omega_i\left(-\ln\left(1-\sigma_i(x)\right)\right)^{\text{₵}}\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)+\left(q\left(-\ln\left(1-e^{-\left(\omega_j\left(-\ln\left(1-\sigma_j(x)\right)\right)^{\text{₵}}\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)^{1/\text{₵}}}, \\ 1-e^{-\left(\wp\left(-\ln\left(1-e^{-\left(\text{Б}\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(1-e^{-\left(\omega_i\left(-\ln\left(v_i(x)\right)\right)^{\text{₵}}\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)+\left(q\left(-\ln\left(1-e^{-\left(\omega_j\left(-\ln\left(v_j(x)\right)\right)^{\text{₵}}\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)\right)^{1/\text{₵}}}\right)\right)^{\text{₵}}\right)^{1/\text{₵}}} \end{array}\right). \tag{21}$$

$\square$

Example 3. Consider $\Gamma_1 = (0.15, 0.42), \Gamma_2 = (0.37, 0.58)$ and $\Gamma_3 = (0.62, 0.17)$ be three IFVs with weight vector $\varpi = (0.25, 0.40, 0.35)$ and $p = 2, q = 1$, ₲ $= 3$. Then, we aggregate the value of IFAAWHM as also an IFV for $\wp = 1/3$ and Б $= 1/6$. So, we have

$$\begin{aligned}&\mathrm{IFAAWHM}^{p,q}(\Gamma_1,\Gamma_2,\Gamma_3)\\&=\left(e^{-\left(1/3\left(-\ln\left(1-e^{-\left(1/6\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\left(-\ln\left(1-e^{-\left(\left(\left(2\left(-\ln\left(1-e^{-\left(\varpi_i\left(-\ln\left(1-\sigma_i(x)\right)\right)^3\right)^{1/3}}\right)\right)\right)^3\right)+\left(\left(1\left(-\ln\left(1-e^{-\left(\varpi_j\left(-\ln\left(1-\sigma_j(x)\right)\right)^3\right)^{1/3}}\right)\right)\right)^3\right)\right)^{1/3}}\right)\right)\right)^3\right)^{1/3}}\right)\right)^3\right)^{1/3}},\right.\\&\left.1-e^{-\left(1/3\left(-\ln\left(1-e^{-\left(1/6\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\left(-\ln\left(1-e^{-\left(\left(\left(2\left(-\ln\left(1-e^{-\left(\varpi_i\left(-\ln\left(v_i(x)\right)\right)^3\right)^{1/3}}\right)\right)\right)^3\right)+\left(\left(1\left(-\ln\left(1-e^{-\left(\varpi_j\left(-\ln\left(v_j(x)\right)\right)^3\right)^{1/3}}\right)\right)\right)^3\right)\right)^{1/3}}\right)\right)\right)^3\right)^{1/3}}\right)\right)^3\right)^{1/3}}\right)\\&=(0.3616, 0.4466).\end{aligned} \tag{22}$$

Theorem 6 (Idempotency property). *Consider* $\Gamma_i = (v_{\Gamma_i}(x), v_{\Gamma_i}(x)) = \Gamma = (v_\Gamma(x), v_\Gamma(x)), i = 1, 2, 3, \ldots, и$ *are identical IFVs. Then,* $\mathrm{IFAAWHM}^{p,q}(\Gamma_1, \Gamma_2, \Gamma_3, \ldots, \Gamma_и) = \Gamma$.

Proof. Proof is straightforward. □

Theorem 7 (Monotonicity). *Consider* $\Gamma_i = (\sigma_{\Gamma_i}(x), v_{\Gamma_i}(x))$ *and* $\varsigma_i = (\sigma_{\varsigma_i}(x), v_{\varsigma_i}(x)), i = 1, 2, 3, \ldots, и$ *to be any two IFSs, if* $\Gamma_i \le \varsigma_i, v_{\Gamma_i}(x) \ge v_{\varsigma_i}(x), \forall, i$. *Then,* $\mathrm{IFAAWHM}^{p,q}(\Gamma_1, \Gamma_2, \Gamma_3, \ldots, \Gamma_и) \le \mathrm{IFAAWHM}^{p,q}(\varsigma_1, \varsigma_2, \varsigma_3, \ldots, \varsigma_и)$.

Proof. Proof is analogous. □

Theorem 8 (Boundedness). *Consider* $\Gamma_i = (\sigma_{\Gamma_i}(x), v_{\Gamma_i}(x))$, $i = 1, 2, 3, \ldots, и$ *to be an IFSs, and* $\Gamma_i^- = \min(\Gamma_1, \Gamma_2, \Gamma_3, \ldots, \Gamma_и)$ *and* $\Gamma_i^+ = \max(\Gamma_1, \Gamma_2, \Gamma_3, \ldots, \Gamma_и)$, *then,*

$$\Gamma_i^- \le \mathrm{IFAAWHM}^{p,q}(\Gamma_1, \Gamma_2, \Gamma_3, \ldots, \Gamma_и) \le \Gamma_i^+. \tag{23}$$

5. Intuitionistic Fuzzy Aczel Alsina Geometric Heronian Mean and Weighted Heronian Mean Operators

Now, we generalized the theory of the GHM operator by using the Aczel Alsina operations under the system IFSs. We gave an example to check the competitiveness of AOs.

Definition 10. Consider $\Gamma_i = (\sigma_{\Gamma_i}, v_{\Gamma_i}), i = 1, 2, 3, \ldots, и$ be the family of IFVs and $p, q \ge 0$, then, IFAAGHM operator is defined as follows:

$$\mathrm{IFAAGHM}(\Gamma_1, \Gamma_2, \ldots, \Gamma_и) == \frac{1}{p+q} \underset{i=1}{\overset{и}{\otimes}} \underset{j=1}{\overset{и}{\otimes}} \left(p\Gamma_i \oplus q\Gamma_j\right)^{2/и(и+1)}. \tag{24}$$

Theorem 9. *Consider* $\Gamma_i = (\sigma_{\Gamma_i}, v_{\Gamma_i}), i = 1, 2, 3, \ldots, и$ *be the family of IFVs and* $p, q \ge 0$, *then, aggregated value of the IFAAGHM operator is defined as follows:*

$$\mathrm{IFAAGHM}\left(\Gamma_1,\Gamma_2,\ldots,\Gamma_n\right) = \left(\begin{array}{c} 1-e^{-\left((1/p+q)\left(-\ln\left(1-e^{-\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left((2/\text{и}(\text{и}+1))\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(1-\sigma_i(x)\right)\right)^{\mathbb{G}}\right)+\left(q\left(-\ln\left(1-\sigma_j(x)\right)\right)^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}}\right)\right)\right)^{\mathbb{G}}\right)^{1/\mathbb{G}}}\right)\right)\right)^{\mathbb{G}}\right)^{1/\mathbb{G}}}, \\ e^{-\left((1/p+q)\left(-\ln\left(1-e^{-\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left((2/\text{и}(\text{и}+1))\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(v_i(x)\right)\right)^{\mathbb{G}}\right)+\left(q\left(-\ln\left(v_j(x)\right)\right)^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}}\right)\right)\right)^{\mathbb{G}}\right)^{1/\mathbb{G}}}\right)\right)\right)^{\mathbb{G}}\right)^{1/\mathbb{G}}} \end{array}\right). \tag{25}$$

Proof. Consider the $\Gamma_i = (\sigma_{\Gamma_i}\, v_{\Gamma_i}), i = 1,2,3,\ldots,\text{и}$, We have:

$$p\Gamma_i = \left(1-e^{-\left(p\left(-\ln\left(1-\sigma_i(x)\right)\right)^{\mathbb{G}}\right)^{1/\mathbb{G}}}, e^{-\left(p\left(-\ln\left(v_i(x)\right)\right)^{\mathbb{G}}\right)^{1/\mathbb{G}}}\right),$$

$$q\Gamma_j = \left(1-e^{-\left(q\left(-\ln\left(1-\sigma_j(x)\right)\right)^{\mathbb{G}}\right)^{1/\mathbb{G}}}, e^{-\left(q\left(-\ln\left(v_j(x)\right)\right)^{\mathbb{G}}\right)^{1/\mathbb{G}}}\right),$$

$$\left(p\Gamma_i \oplus q\Gamma_j\right) = \left(\begin{array}{c} 1-e^{-\left(\left(p\left(-\ln\left(1-\sigma_i(x)\right)\right)^{\mathbb{G}}\right)+\left(q\left(-\ln\left(1-\sigma_j(x)\right)\right)^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}} \\ e^{-\left(\left(p\left(-\ln\left(v_i(x)\right)\right)^{\mathbb{G}}\right)+\left(q\left(-\ln\left(v_j(x)\right)\right)^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}} \end{array}\right),$$

$$\left(p\Gamma_i \oplus q\Gamma_j\right)^{(2/\text{и}(\text{и}+1))} = \left(\begin{array}{c} e^{-\left((2/\text{и}(\text{и}+1))\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(1-\sigma_i(x)\right)\right)^{\mathbb{G}}\right)+\left(q\left(-\ln\left(1-\sigma_j(x)\right)\right)^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}}\right)\right)^{\mathbb{G}}\right)^{1/\mathbb{G}}} \\ 1-e^{-\left((2/\text{и}(\text{и}+1))\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(v_i(x)\right)\right)^{\mathbb{G}}\right)+\left(q\left(-\ln\left(v_j(x)\right)\right)^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}}\right)\right)^{\mathbb{G}}\right)^{1/\mathbb{G}}} \end{array}\right),$$

$$\underset{j=1}{\overset{\text{и}}{\otimes}}\left(p\Gamma_i \oplus q\Gamma_j\right)^{(2/\text{и}(\text{и}+1))} = \left(\begin{array}{c} e^{-\left(\sum_{j=1}^{\text{и}}\left((2/\text{и}(\text{и}+1))\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(1-\sigma_i(x)\right)\right)^{\mathbb{G}}\right)+\left(q\left(-\ln\left(1-\sigma_j(x)\right)\right)^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}}\right)\right)^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}}, \\ 1-e^{-\left(\sum_{j=1}^{\text{и}}\left((2/\text{и}(\text{и}+1))\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(v_i(x)\right)\right)^{\mathbb{G}}\right)+\left(q\left(-\ln\left(v_j(x)\right)\right)^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}}\right)\right)^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}} \end{array}\right),$$

$$\underset{i=1}{\overset{\text{и}}{\otimes}}\underset{j=1}{\overset{\text{и}}{\otimes}}\left(p\Gamma_i \oplus q\Gamma_j\right)^{(2/\text{и}(\text{и}+1))} = \left(\begin{array}{c} e^{-\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left((2/\text{и}(\text{и}+1))\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(1-\sigma_i(x)\right)\right)^{\mathbb{G}}\right)+\left(q\left(-\ln\left(1-\sigma_j(x)\right)\right)^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}}\right)\right)^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}}, \\ 1-e^{-\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left((2/\text{и}(\text{и}+1))\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(v_i(x)\right)\right)^{\mathbb{G}}\right)+\left(q\left(-\ln\left(v_j(x)\right)\right)^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}}\right)\right)^{\mathbb{G}}\right)\right)^{1/\mathbb{G}}} \end{array}\right),$$

$$\frac{1}{p+q}\overset{и}{\underset{i=1}{\otimes}}\overset{и}{\underset{j=1}{\otimes}}\left(p\Gamma_i\oplus q\Gamma_j\right)^{(2/и(и+1))}$$
$$=\left(\begin{array}{c}1-e^{-\left((1/p+q)\left(-\ln\left(1-e^{-\left(\sum_{i=1}^{и}\sum_{j=1}^{и}\left((2/и(и+1))\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(1-\sigma_i(x)\right)\right)^{\mathfrak{G}}\right)+\left(q\left(-\ln\left(1-\sigma_j(x)\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}},\\ e^{-\left((1/p+q)\left(-\ln\left(1-e^{-\left(\sum_{i=1}^{и}\sum_{j=1}^{и}\left((2/и(и+1))\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(v_i(x)\right)\right)^{\mathfrak{G}}\right)+\left(q\left(-\ln\left(v_j(x)\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}\end{array}\right). \tag{26}$$

□

Example 4. Consider $\Gamma_1=(0.34,0.28), \Gamma_2=(0.67,0.44), \Gamma_3=(0.51,0.26)$ be three IFVs, and $p=2, q=1, \mathfrak{G}=3$, then, we aggregate the value of IFAAHM as also an IFV. We have

$$\text{IFAAGHM}^{p,q}\left(\Gamma_1,\Gamma_2,\Gamma_3,\ldots,\Gamma_и\right)$$
$$=\left(\begin{array}{c}1-e^{-\left((1/p+q)\left(-\ln\left(1-e^{-\left(\sum_{i=1}^{и}\sum_{j=1}^{и}\left((2/и(и+1))\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(1-\sigma_i(x)\right)\right)^{\mathfrak{G}}\right)+\left(q\left(-\ln\left(1-\sigma_j(x)\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}},\\ e^{-\left((1/p+q)\left(-\ln\left(1-e^{-\left(\sum_{i=1}^{и}\sum_{j=1}^{и}\left((2/и(и+1))\left(-\ln\left(1-e^{-\left(\left(p\left(-\ln\left(v_i(x)\right)\right)^{\mathfrak{G}}\right)+\left(q\left(-\ln\left(v_j(x)\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}\end{array}\right),$$
$$\text{IFAAGHM}^{2,1}\left(\Gamma_1,\Gamma_2,\Gamma_3\right)=\left(\begin{array}{c}1-e^{-\left((1/3)\left(-\ln\left(1-e^{-\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\left((1/6)\left(-\ln\left(1-e^{-\left(\left(2\left(-\ln\left(1-\sigma_i(x)\right)\right)^{3}\right)+\left(1\left(-\ln\left(1-\sigma_j(x)\right)\right)^{3}\right)\right)^{1/3}}\right)\right)^{3}\right)\right)^{1/3}}\right)\right)^{3}\right)^{1/3}},\\ e^{-\left((1/3)\left(-\ln\left(1-e^{-\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\left((1/6)\left(-\ln\left(1-e^{-\left(\left(2\left(-\ln\left(v_i(x)\right)\right)^{3}\right)+\left(1\left(-\ln\left(v_j(x)\right)\right)^{3}\right)\right)^{1/3}}\right)\right)^{3}\right)\right)^{1/3}}\right)\right)^{3}\right)^{1/3}}\end{array}\right),$$
$$=(0.4598, 0.3453). \tag{27}$$

Remark 2. The basic properties of the IFAAGHM operator like idempotency, monotonicity, and boundedness are similar to the proof of Theorems 2–4.

Definition 11. Consider $\Gamma_i=(\sigma_{\Gamma_i}\, v_{\Gamma_i})$ be the family of IFVs with weight vector $\varpi=(\varpi_1,\varpi_2,\varpi_3,\ldots,\varpi_и)^T, \varpi_и\in[0,1], \sum_{i=1}^{и}\varpi_i=1$, and $p>0, q>0$, then. an the IFAA weighted geometric Heronian mean (IFAAWGHM) is defined as follows:

$$\text{IFAAWGHM}^{p,q}\left(\Gamma_1,\Gamma_2,\Gamma_3,\ldots,\Gamma_{\text{и}}\right)=\frac{1}{p+q}\left(\underset{i=1}{\overset{\text{и}}{\otimes}}\underset{j=1}{\overset{\text{и}}{\otimes}}\left(\left(p\Gamma_i\right)^{\omega_i}\dot{\oplus}\left(q\Gamma_j\right)^{\omega_j}\right)^{2/\text{и}(\text{и}+1)}\right). \tag{28}$$

Theorem 10. *Consider $\Gamma_i=(\sigma_{\Gamma_i}\, v_{\Gamma_i})$ be the family of IFVs with weight vector $\omega=(\omega_1,\omega_2,\omega_3,\ldots,\omega_{\text{и}})^T,\omega_{\text{и}}\in[0,1],\sum_{i=1}^{\text{и}}\omega_i=1,$ and $p>0,q>0,$ then, an intuitionistic fuzzy Aczel Alsina (IFAA) weighted Heronian mean (IFAAWHM) is defined as follows:*

$$\text{IFAAWGHM}^{p,q}\left(\Gamma_1,\Gamma_2,\Gamma_3,\ldots,\Gamma_{\text{и}}\right)=\left(\begin{array}{c}1-e^{-\left(\left((1/p+q)\left(-\ln\left(1-e^{-\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left((2/\text{и}(\text{и}+1))\left(-\ln\left(1-e^{-\left(\alpha_i+\alpha_j\right)^{1/\mathfrak{G}}}\right)\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}\right)\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}},\\ e^{-\left(\left((1/p+q)\left(-\ln\left(1-e^{-\left(\sum_{i=1}^{\text{и}}\sum_{j=1}^{\text{и}}\left((2/\text{и}(\text{и}+1))\left(-\ln\left(1-e^{-\left(\beta_i+\beta_j\right)^{1/\mathfrak{G}}}\right)\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}\right)\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}\end{array}\right), \tag{29}$$

where

$$\begin{aligned}\alpha_i&=\left(-\ln\left(1-e^{-\left(\omega_i\left(-\ln\left(1-e^{-\left(p\left(-\ln\left(1-\sigma_i(x)\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}},\\ \alpha_j&=\left(-\ln\left(1-e^{-\left(\omega_j\left(-\ln\left(1-e^{-\left(q\left(-\ln\left(1-\sigma_j(x)\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}},\\ \beta_i&=\left(-\ln\left(1-e^{-\left(\omega_i\left(-\ln\left(1-e^{-\left(p\left(-\ln\left(v_i(x)\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}},\\ \beta_j&=\left(-\ln\left(1-e^{-\left(\omega_j\left(-\ln\left(1-e^{-\left(q\left(-\ln\left(v_j(x)\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}.\end{aligned} \tag{30}$$

Proof. Prove of this theorem is similar to Theorem 5. ☐

Example 5. Consider $\Gamma_1 = (0.17, 0.52), \Gamma_2 = (0.27, 0.62), \Gamma_3 = (0.32, 0.57)$ be three IFVs with weight vector $\omega = (0.45, 0.25, 0.30)$, and $p = 2, q = 1, \mathfrak{G} = 3$, then, we aggregate the value of IFAAWHM as also an IFV. We have

$$\text{IFAAWGHM}^{2,1}(\Gamma_1,\Gamma_2,\Gamma_3) = \left(1 - e^{-\left((1/3)\left(-\ln\left(1-e^{-\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\left((1/6)\left(-\ln\left(1-e^{-(\alpha_i+\alpha_j)^{1/3}}\right)\right)\right)^{3}\right)^{1/3}}\right)\right)^{3}\right)^{1/3}}, e^{-\left((1/3)\left(-\ln\left(1-e^{-\left(\sum_{i=1}^{3}\sum_{j=1}^{3}\left((1/6)\left(-\ln\left(1-e^{-(\beta_i+\beta_j)^{1/3}}\right)\right)\right)^{3}\right)^{1/3}}\right)\right)^{3}\right)^{1/3}} \right), \tag{31}$$

where

$$\begin{aligned}
\alpha_i &= \left(-\ln\left(1-e^{-\left(\omega_i\left(-\ln\left(1-e^{-\left(p\left(-\ln\left(1-\sigma_i(x)\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}},\\
\alpha_j &= \left(-\ln\left(1-e^{-\left(\omega_j\left(-\ln\left(1-e^{-\left(q\left(-\ln\left(1-\sigma_j(x)\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}},\\
\beta_i &= \left(-\ln\left(1-e^{-\left(\omega_i\left(-\ln\left(1-e^{-\left(p\left(-\ln\left(\upsilon_i(x)\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}},\\
\beta_j &= \left(-\ln\left(1-e^{-\left(\omega_j\left(-\ln\left(1-e^{-\left(q\left(-\ln\left(\upsilon_j(x)\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}}\right)^{1/\mathfrak{G}}}\right)\right)^{\mathfrak{G}},\\
&= (0.3538, 0.4500).
\end{aligned} \tag{32}$$

In this section, we evaluate the flexibility, and competency of our proposed methodology by using MADM techniques based on the IF information.

6. MADM Techniques Based on Intuitionistic Fuzzy Information

A MADM technique is a process to select a suitable alternative from the set of alternatives based on some attributes defined by the decision-maker. Consider a finite set of alternatives $\Lambda = (\Lambda_1, \Lambda_2, \ldots, \Lambda_n)$, and the attributes are denoted by $\Xi = (\Xi_1, \Xi_2, \ldots, \Xi_n)$ defined by the decision-maker. The set of finite alternatives based on attributes closed in $[0,1]$ in the environment of IFVs. Let $£ = (£_1, £_2, \ldots, £_n)$ be the set of weight vectors, $£_i \in [0,1]$, and $\sum_{i=1}^{и} £_i = 1, i = (1, 2, 3, \ldots, и)$. Consider a decision matrix $A = (\sigma, \upsilon)_{\mathfrak{s}\times\varrho}$ provide from the decision-maker. Each pair $(\sigma_{\mathfrak{s}\varrho}, \upsilon_{\mathfrak{s}\varrho})$ denotes the IFVs, and it must satisfy the condition of $0 \le \sigma + \upsilon \le 1$.

$$A = (\sigma, \upsilon)_{\mathfrak{s}\times\varrho} = \begin{bmatrix} (\sigma_{11}, \upsilon_{11}) & (\sigma_{12}, \upsilon_{12}) & \ldots & (\sigma_{1\varrho}, \upsilon_{1\varrho}) \\ (\sigma_{21}, \upsilon_{21}) & (\sigma_{22}, \upsilon_{22}) & \ldots & (\sigma_{2\varrho}, \upsilon_{2\varrho}) \\ \vdots & \vdots & \ddots & \vdots \\ (\sigma_{\mathfrak{s}1}, \upsilon_{\mathfrak{s}1}) & (\sigma_{\mathfrak{s}2}, \upsilon_{\mathfrak{s}2}) & \ldots & (\sigma_{\mathfrak{s}\varrho}, \upsilon_{\mathfrak{s}\varrho}) \end{bmatrix}. \tag{33}$$

We illustrate the most suitable alternative by utilizing the AOs of IFAAWHM, and the IFAAWGH operators based on the technique of MADM under the environment of the IF information. To evaluate intuitionistic fuzzy information under a MADM technique by using the following steps of the algorithm and we also explored the steps of the algorithm in the following follow chart shown in Figure 1.

Step 1: The decision-maker collects information in the form of IFVs and shows all given information in the form of decision matrices.

Step 2: We must transform the given decision matrix $\underset{\sim}{A}=(\sigma,\upsilon)_{\mathfrak{s}\times\varrho}$ into the normalization matrix $\overline{\underset{\sim}{A}}=\overline{(\sigma,\upsilon)}_{\mathfrak{s}\times\varrho}$ If different types of attributes are involved in the set of attributes like benefit types and cost types. Otherwise, there is no need to transform the decision matrix into a normalization matrix.

Step 3: Applied our robust proposed methodologies such as IFAAWHM and IFAAWGHM operators to determine the optimal option (alternatives from the finite set of alternatives) $\Lambda=(\Lambda_1,\Lambda_2,\ldots,\Lambda_n)$ based on certain characteristics.

Step 4: Compute score values by utilizing the obtained consequences of the IFAAWHM and IFAAWGHM operators.

$$\hat{S}(\Gamma)=\sigma_{\Gamma}(\mathscr{x})-\upsilon_{\Gamma}(\mathscr{x})\in[-1,1]. \tag{34}$$

Step 5: After computation of the score values, in order to investigate more suitable alternative, rearrange score values by the ranking and ordering process.

6.1. Application. Researchers are still interested in learning more about how energy directly influences the economic growth. Research has demonstrated that there is a strong connection between health and economic growth, as compared to the role of energy in economic progress. Energy is a fundamental component of the economic growth. The demand for energy rises in tandem with increasing industrial and agricultural activity. Some have proposed that providing nations with more access to energy will help them build their economy and enhance the lives of the poor. Solar energy is the only remaining option for us to deal with the level of breakdowns and power failures that our country has to encounter. People's focus has turned to the country's advantages in solar energy as a result of the country's current challenges with water, electricity, and pollution our country's geographic location is suitable for solar energy power generation since there is abundant sunlight throughout the year. All year, the sun warms the earth's surface. Our country's solar energy system is a cheap and accessible source of power.

6.2. Numerical Example. In order to support its industrial, agriculture, commercial, and residential sectors, any country needs an adequate supply of solid energy. One main contributing factor to the countries in the development stage is a power shortage. To handle a critical situation of electricity breakdown, a public hospital must install its own energy production system. In this numerical example, a hospital wants to purchase solar panels to enhance its energy resources. Solar panels should be reliable and safe for electronic devices. Several companies manufacture and install their solar panels. The selection of suitable solar panels from different sets of solar panels $\Omega_i, i=(1,2,3,4,5)$ based on defined prominent characteristics $\beta_j, j=(1,2,3,4,)$. β_1: production material (monocrystalline and polycrystalline materials), β_2 dented the quality of the solar panel, β_3 represents the performance in low sunlight, and β_3 represents the cost of solar panels. The corresponding weight vectors $\omega=(0.20,0.05,0.30,0.45)$ of the alternative assumed hypothetically to aggregate information given by the decision-maker is shown in Table 2.

6.3. The Evaluation Process of the MADM Technique

Step 1: Information about solar panels assumed hypothetically in the form of IFVs by the decision maker and is shown in Table 2.

Step 2: There are two types of attributes such that benefit type and cost type, we see $\beta_i, i=1,2,3$ and β_4 are benefit and cost types, respectively. After the transformation of the standard decision matrix in Table 2 into the normalizer matrix in Table 3.

Step 3: Applied our proposed methodologies IFAAWHM and IFAAWGHM operators on the decision matrix Table 3. All obtained results from the proposed approaches are depicted in Table 4 for $\mathfrak{C}=\mathbf{3}$.

Step 4: We computed the score values by using Definition 3. Investigated results of the score values are shown in Table 5.

Step 5: To investigate a more suitable alternative (solar panels), we perform ranking and ordering. After evaluation of the score values, we see Π_4 is a suitable alternative for the IFAAWHM and IFAAWGHM operators. The results of the score values of the IFAAWHM and IFAAWGM operators are displayed as a graphical representation in Figure 2.

6.4. Impact of Parameter on Our Proposed Methodology by the Variation of $\mathfrak{C}$. To explore the impact of parametric values on our discussed technique by the variation of parameter $\mathfrak{C}$. As the magnitude of the parametric value $\mathfrak{C}$, the consequences of the IFAAWHM gradually increases. Similarly, if we increase the magnitude of the parametric value $\mathfrak{C}$, obtained results by the IFAAWGHM operator begin to decrease. All results obtained by the IFAAWHM and IFAAWGHM operators are shown in Tables 6 and 7, respectively. After the evaluation of score values by the raking and ordering process, we observed that the raking of the score values remains the same. These characteristics of the score values show the isotonicity property, which determines if we change the parametric values raking and ordering of the score values remain the same.

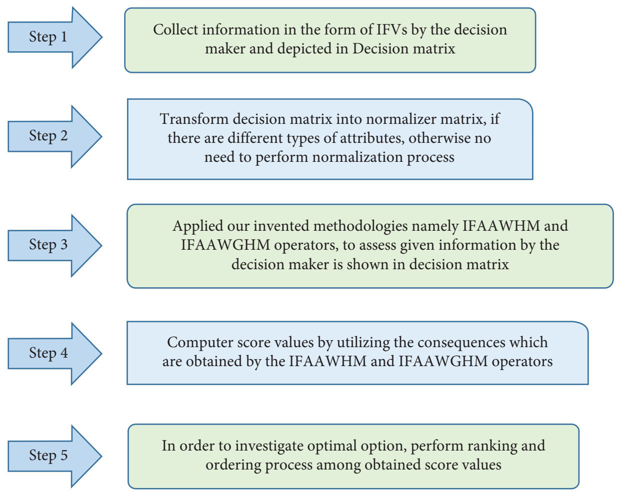

Figure 1: Flowchart of the algorithm.

Table 2: Decision matrix.

	β_1	β_2	β_3	β_4
Π_1	$(0.34, 0.56)$	$(0.12, 0.78)$	$(0.25, 0.47)$	$(0.72, 0.05)$
Π_2	$(0.18, 0.63)$	$(0.22, 0.56)$	$(0.24, 0.36)$	$(0.33, 0.62)$
Π_3	$(0.19, 0.67)$	$(0.36, 0.19)$	$(0.45, 0.01)$	$(0.67, 0.09)$
Π_4	$(0.34, 0.39)$	$(0.38, 0.52)$	$(0.45, 0.11)$	$(0.07, 0.78)$
Π_5	$(0.07, 0.18)$	$(0.09, 0.15)$	$(0.47, 0.13)$	$(0.35, 0.32)$

Table 3: Normalized decision matrix.

	β_1	β_2	β_3	β_4
Π_1	$(0.34, 0.56)$	$(0.12, 0.78)$	$(0.25, 0.47)$	$(0.05, 0.72)$
Π_2	$(0.18, 0.63)$	$(0.22, 0.56)$	$(0.24, 0.36)$	$(0.62, 0.33)$
Π_3	$(0.19, 0.67)$	$(0.36, 0.19)$	$(0.45, 0.01)$	$(0.09, 0.67)$
Π_4	$(0.34, 0.39)$	$(0.38, 0.52)$	$(0.45, 0.11)$	$(0.78, 0.07)$
Π_5	$(0.07, 0.18)$	$(0.09, 0.15)$	$(0.47, 0.13)$	$(0.32, 0.35)$

Table 4: Consequences of the IFAAWHM and IFAAWGHM operators at $\mathfrak{G} = \mathbf{3}$.

	$\Upsilon(\Pi_1)$	$\Upsilon(\Pi_2)$	$\Upsilon(\Pi_3)$	$\Upsilon(\Pi_4)$	$\Upsilon(\Pi_5)$
IFAAWHM	$(0.1540, 0.7141)$	$(0.3369, 0.5695)$	$(0.2087, 0.2043)$	$(0.4719, 0.2861)$	$(0.2441, 0.3756)$
IFAAWGHM	$(0.2740, 0.4776)$	$(0.4515, 0.3425)$	$(0.3410, 0.4305)$	$(0.6136, 0.2027)$	$(0.3514, 0.1789)$

Table 5: Score values of the IFAAWHM and IFAAWGHM operators.

	Sco Π_1	Sco Π_2	Sco Π_3	Sco Π_4	Sco Π_5	Ranking and ordering
IFAAWHM	-0.5601	-0.2326	0.0045	0.1857	-0.1342	$\Pi_4 \succ \Pi_3 \succ \Pi_5 \succ \Pi_2 \succ \Pi_1$
IFAAWGHM	-0.2037	0.1090	-0.0895	0.4109	0.1725	$\Pi_4 \succ \Pi_5 \succ \Pi_2 \succ \Pi_3 \succ \Pi_1$

Figure 3 graphical expression determines the results of different parametric values $\mathfrak{G}$, which are depicted in Table 6 and obtained by the IFAAWHM operator.

In Figure 4, graphical expression determine the results of different parametric values $\mathfrak{G}$ which are depicted in Table 7 and obtained by the IFAAWGHM operator.

7. Comparative Analysis

Several researchers utilized the concepts of the MADM technique to find the reliability and validity of our discussed methodology, and we compared some existing AOs with our developing AOs. Some AOs presented by Yu [40], Xu [9],

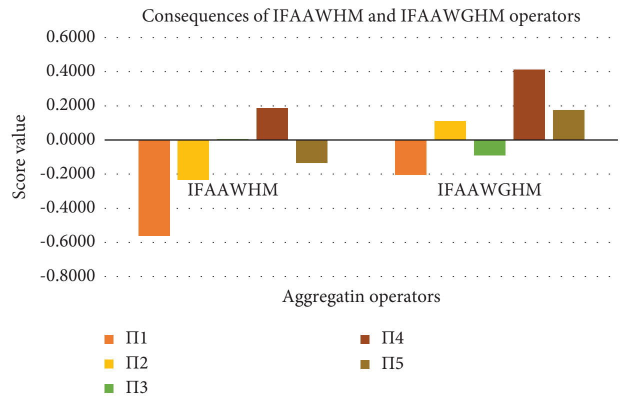

FIGURE 2: Graphical representation of the outcomes of the IFAAWHM and IFAAWGHM operators.

TABLE 6: Consequences of the score values of the IFAAWHM by the variations of $\mathfrak{G}$.

	Sco Π_1	Sco Π_2	Sco Π_3	Sco Π_4	Sco Π_5	Ranking and ordering
$\mathfrak{G}=1$	−0.8115	−0.6053	−0.5153	−0.3354	−0.5627	$\Pi_4>\Pi_3>\Pi_5>\Pi_2>\Pi_1$
$\mathfrak{G}=3$	−0.5601	−0.2326	0.0045	0.1857	−0.1342	$\Pi_4>\Pi_3>\Pi_5>\Pi_2>\Pi_1$
$\mathfrak{G}=7$	−0.3881	0.0192	0.2551	0.4685	0.0792	$\Pi_4>\Pi_3>\Pi_5>\Pi_2>\Pi_1$
$\mathfrak{G}=35$	−0.1939	0.2267	0.4027	0.6626	0.2800	$\Pi_4>\Pi_3>\Pi_5>\Pi_2>\Pi_1$
$\mathfrak{G}=65$	−0.1650	0.2553	0.4199	0.6848	0.3075	$\Pi_4>\Pi_3>\Pi_5>\Pi_2>\Pi_1$
$\mathfrak{G}=85$	−0.1569	0.2634	0.4246	0.6908	0.3152	$\Pi_4>\Pi_3>\Pi_5>\Pi_2>\Pi_1$
$\mathfrak{G}=101$	−0.1527	0.2676	0.4270	0.6939	0.3191	$\Pi_4>\Pi_3>\Pi_5>\Pi_2>\Pi_1$
$\mathfrak{G}=115$	−0.1499	0.2703	0.4286	0.6959	0.3216	$\Pi_4>\Pi_3>\Pi_5>\Pi_2>\Pi_1$
$\mathfrak{G}=135$	−0.1470	0.2732	0.4303	0.6980	0.3244	$\Pi_4>\Pi_3>\Pi_5>\Pi_2>\Pi_1$
$\mathfrak{G}=153$	−0.1450	0.2752	0.4314	0.6994	0.3262	$\Pi_4>\Pi_3>\Pi_5>\Pi_2>\Pi_1$

TABLE 7: The consequences of the score values of the IFAAWGHM by the variations of $\mathfrak{G}$.

	Sco Π_1	Sco Π_2	Sco Π_3	Sco Π_4	Sco Π_5	Ranking and ordering
$\mathfrak{G}=1$	0.2251	0.4768	0.3183	0.6873	0.4991	$\Pi_4>\Pi_5>\Pi_2>\Pi_3>\Pi_1$
$\mathfrak{G}=3$	−0.2037	0.1090	−0.0895	0.4109	0.1725	$\Pi_4>\Pi_5>\Pi_2>\Pi_3>\Pi_1$
$\mathfrak{G}=7$	−0.4621	−0.1354	−0.3493	0.1790	−0.0531	$\Pi_4>\Pi_5>\Pi_2>\Pi_3>\Pi_1$
$\mathfrak{G}=35$	−0.6646	−0.3743	−0.5348	−0.0882	−0.2338	$\Pi_4>\Pi_5>\Pi_2>\Pi_3>\Pi_1$
$\mathfrak{G}=65$	−0.6950	−0.4092	−0.5558	−0.1299	−0.2552	$\Pi_4>\Pi_5>\Pi_2>\Pi_3>\Pi_1$
$\mathfrak{G}=85$	−0.7034	−0.4188	−0.5616	−0.1415	−0.2611	$\Pi_4>\Pi_5>\Pi_2>\Pi_3>\Pi_1$
$\mathfrak{G}=101$	−0.7076	−0.4237	−0.5645	−0.1476	−0.2641	$\Pi_4>\Pi_5>\Pi_2>\Pi_3>\Pi_1$
$\mathfrak{G}=115$	−0.7104	−0.4269	−0.5664	−0.1515	−0.2661	$\Pi_4>\Pi_5>\Pi_2>\Pi_3>\Pi_1$
$\mathfrak{G}=135$	−0.7133	−0.4304	−0.5684	−0.1557	−0.2681	$\Pi_4>\Pi_5>\Pi_2>\Pi_3>\Pi_1$
$\mathfrak{G}=153$	−0.7153	−0.4327	−0.5698	−0.1585	−0.2695	$\Pi_4>\Pi_5>\Pi_2>\Pi_3>\Pi_1$

Aczel Alsina AOs given by Senapati et al. [49], Xu and Yager [11], Seikh and Mandal [50], Wang and Liu [51], Wang and Liu [52], Senapati et al. [49], Wu et al. [53], and Wang et al. [54]. We carefully applied the previously discussed AOs to the information shown in the decision matrix in Table 2. After evaluating information by utilizing the existing AOs, the consequences of existing AOs are shown in the following Table 8. AOs of IVIFDHM operators presented by Wu et al. [53] and AOs of IVIFWA given by Wang et al. [54] failed to aggregate information given by the decision matrix. The following table shows the ranking of the results of AOs of IFWA, IFWG, IFDWA, IFDWG, IFEWA, IFEWG, IFAAWA, and IFIFAAWG operators 8.

From the results of the existing approaches determined in Table 8, we observed that our invented methodologies are more reliable and efficient than other aggregation models. The geometrical representation of Figure 5 expressed the results of a comparative study shown in Table 8.

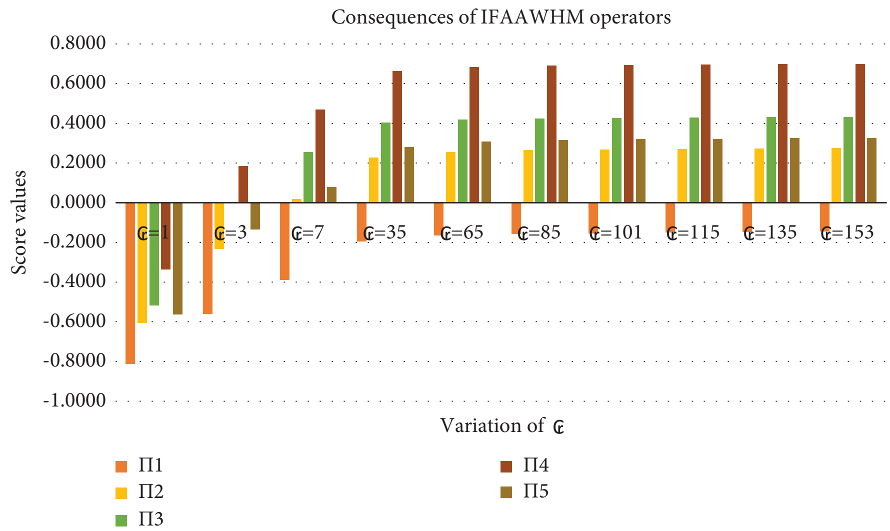

Figure 3: Graphical representation of the IFAAWHM by the variations of 𝕰.

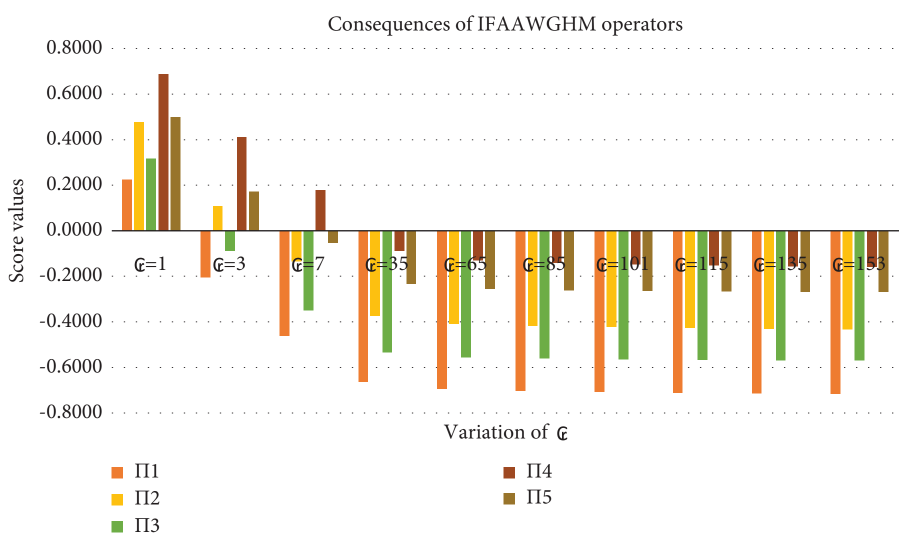

Figure 4: Graphical representation of the IFAAWGHM by the variations of 𝕰.

Table 8: The comparison of discussed AOs with existing AOs.

Aggregation operators		Ranking and ordering
Current discussed AO	IFAAWHM	$\Pi_4 \succ \Pi_3 \succ \Pi_5 \succ \Pi_2 \succ \Pi_1$
Current discussed AO	IFAAWGHM	$\Pi_4 \succ \Pi_5 \succ \Pi_2 \succ \Pi_3 \succ \Pi_1$
Yu [40]	IFGWHM	$\Pi_4 \succ \Pi_2 \succ \Pi_5 \succ \Pi_3 \succ \Pi_1$
Xu [9]	IFWA	$\Pi_2 \succ \Pi_1 \succ \Pi_4 \succ \Pi_5 \succ \Pi_3$
Xu and Yager [11]	IFWG	$\Pi_4 \succ \Pi_5 \succ \Pi_2 \succ \Pi_3 \succ \Pi_1$
Seikh and Mandal [50]	IFDWA	$\Pi_4 \succ \Pi_3 \succ \Pi_5 \succ \Pi_2 \succ \Pi_1$
Seikh and Mandal [50]	IFDWG	$\Pi_4 \succ \Pi_5 \succ \Pi_2 \succ \Pi_3 \succ \Pi_1$
Wang and Liu [51]	IFEWA	$\Pi_4 \succ \Pi_5 \succ \Pi_3 \succ \Pi_2 \succ \Pi_1$
Wang and Liu [52]	IFEWG	$\Pi_4 \succ \Pi_2 \succ \Pi_5 \succ \Pi_3 \succ \Pi_1$
Senapati et al. [49]	IFAAWA	$\Pi_4 \succ \Pi_5 \succ \Pi_3 \succ \Pi_2 \succ \Pi_1$
Senapati et al. [49]	IFAAWG	$\Pi_4 \succ \Pi_5 \succ \Pi_2 \succ \Pi_3 \succ \Pi_1$
Wu et al. [53]	IVIFDHM	Failed
Wang et al. [54]	IVIFWA	Failed

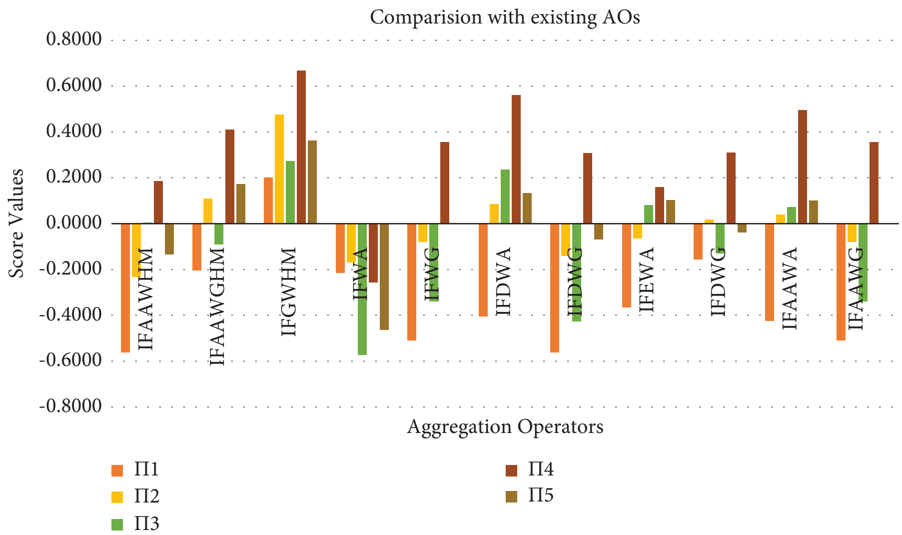

Figure 5: The comparison of the consequences of existing AOs with discussed AOs.

8. Conclusion

A subfield of operations research known as the MADM technique focuses on finding the best solution by systematically evaluating the possible answers against many options. In many fields, including social science, economics, and medical science, MADM challenges are encountered and extensively used. Aggregation models attained a lot of attention in the field of research by numerous research scientists. The AOs play a substantial role in assessing optimal solutions under the MADM technique. The main goal of this article is to handle uncertain and ambiguous information under the IF information system. We studied concepts of triangular norms and their generalization in the form of robust aggregation tools like Aczel Alsina operations. We explored the theories of HM and GHM tools to overcome insufficient information and define strong interrelationships among different arguments. We developed a list of certain AOs of HM tools by utilizing the Aczel Alsina operations under the system of IF information, namely, IFAAHM and IFAAWHM operators.

Moreover, we also extended the theory of GHM tools based on Aczel Alsina operations and developed a new series of AOs in the form of IFAAGHM and IFAAWGHM operators. Some specific properties of our invented approaches are also presented. We applied our conceived method of IFAAWHM and IFAAWGHM operators to choose optimal options under the system of the MADM technique. A numerical example was also given to select more suitable solar panels under our proposed methodologies. To find the competitiveness and feasibility of discussed methods, we effectively compare the existing approaches with the results of the currently proposed processes. However, our proposed procedures are more reliable. They have the extensive capacity to deal with real-life problems like medical diagnosis, enterprise resource management systems, energy crises, construction management, and a military academy's selection process.

The method's shortcoming is that it is unable to handle situations involving many types of decision information. The characteristics of set-theoretic operations and logical outcomes of the expanded PyFSs, as well as other relevant notions, will be studied in future research as a practical tool to handle dubious information and related ideas. To address decision-making issues with unique properties, we shall further investigate picture fuzzy sets [55] and bipolar soft sets [56, 57]. In addition, we will use a recently created algorithm to address more difficult decision-making situations [58–60].

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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