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RESEARCH ARTICLE-MECHANICAL ENGINEERING

# Damage Factor Calculation for Condition Monitoring of Rolling Bearings

**Ivana D. Atanasovska**[1] · **Natasa D. Soldat**[2] · **Santosh S. Patil**[3] · **Radivoje M. Mitrovic**[2] · **Radoslav Z. Tomovic**[4]

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## Abstract

New demands placed on industry make it necessary to improve design, real-time monitoring and failure prediction methods for rolling bearings. This paper presents an approach for assessing how contact surface damage affects the vibration response of rolling bearings and how vibration-based monitoring shows the damage level of rolling element bearings. A new parameter named the damage factor is introduced to quantify the vibration response of rolling bearings. The analytical–numerical method for calculation of this parameter is developed based on nonlinear dynamics postulates. The developed method for damage factor calculation is tested on a particular type of radial deep groove ball bearing, and an experimental verification is performed, showing a maximum deviation of 7% in the results. The conclusions drawn from the obtained results and the potential contributions of the introduced parameters for rolling bearing design are discussed. The chosen polynomial function keeps deviations in relation to the calculated damage factor within 3% for failed areas of different sizes.

**Keywords** Rolling bearings · Damage factor · Vibration measurement · Nonlinear dynamics · Finite element analysis

## List of Symbols

| | |
|---|---|
| $A_d$ | Maximum vibration amplitude of a damaged bearing, m/s$^2$ |
| $A_u$ | Vibration amplitude of an undamaged bearing, m/s$^2$ |
| $B$ | Bearing width, m |
| $C$ | Dynamic load capacity, N |
| $C_0$ | Static load capacity, N |
| $\mathbf{C}(t)$ | Damping matrix |
| $C_r$ | Radial deep groove ball bearing damping, N s/m |
| $D$ | Bearing outer race diameter, m |
| $d$ | Bearing inner race diameter, m |
| $d_0$ | Average diameter of damages for particular rolling bearing type (obtained empirically), m |
| $d_b$ | Diameter of bearing rolling elements, m |
| $d_c$ | Diameter of the dividing axis of the bearing cage, m |
| $d_d$ | Diameter of damage on a contact surface, m |
| $d_i$ | Diameter of the inner race, m |
| $d_o$ | Diameter of the outer race, m |
| $E$ | Young's modulus, N/m$^2$ |
| $\mathbf{F}(t)$ | Vector of external forces |
| $F$ | External load, N |
| $F_\delta$ | Load distributed on the most loaded rolling element, N |
| $f_i$ | Specific inner ring frequency, Hz |
| $\mathbf{K}(t)$ | Stiffness matrix |
| $K_r$ | Radial deep grove ball bearing stiffness, N/m |
| $K_V$ | Damage factor |

✉ Ivana D. Atanasovska
iatanasovska@mi.sanu.ac.rs; iviatanasov@yahoo.com

Natasa D. Soldat
nsoldat@mas.bg.ac.rs

Santosh S. Patil
santosh.patil@jaipur.manipal.edu; santosh045@gmail.com

Radivoje M. Mitrovic
rmitrovic@mas.bg.ac.rs

Radoslav Z. Tomovic
radoslav@ucg.ac.me

1 Mathematical Institute of the Serbian Academy of Sciences and Arts, Kneza Mihaila 36, 11000 Belgrade, Serbia

2 Faculty of Mechanical Engineering, University of Belgrade, Kraljice Marije 16, 11000 Belgrade, Serbia

3 Department of Mechanical Engineering, Manipal University Jaipur, Jaipur, India

4 Faculty of Mechanical Engineering, University of Montenegro, Dzordza Vasingtona bb, 81000 Podgorica, Montenegro

| | |
|---|---|
| $\mathbf{M}(t)$ | Mass matrix |
| $m_{red}$ | Reduced mass of system shaft-bearing-casing, kg |
| $n$ | Rotational velocity, rpm |
| $\mathbf{Q}(t)$ | Vector of nonlinearity coefficient |
| $q$ | Nonlinearity coefficient, N/m$^3$ |
| $r_{ti}$ | Inner ring groove radius, m |
| $r_{to}$ | Outer ring groove radius, m |
| $t$ | Time, s |
| $\mathbf{y}(t)$ | Vector of the generalized displacement |
| $y$ | Displacement in radial direction, m |
| $z$ | Total number of rolling elements |
| $\alpha$ | Bearing contact angle, degrees |
| $\gamma$ | Angular distance between rolling elements, degrees |
| $\delta$ | Local contact deformation on the most loaded rolling element and raceway surface with damage, m |
| $\varepsilon$ | Total radial displacement of axis of bearing in radial direction, m |
| $\mu$ | Friction coefficient in contact zones |
| $\mu_m$ | Poisson's ratio |
| $\omega$ | Angular velocity, s$^{-1}$ |

## 1 Introduction

New trends in the machine industry cover a wide range of improvements in manufacturing processes and operation conditions of complex mechanical systems and industry plants, which enable complete manufacturing digitalization and computerization [1, 2]. As a result, comprehensive research is needed to adapt mechanical design to new ideas [3, 4]. Several requirements have to be fulfilled to model rotating machinery. These requirements include strict consideration of the reliability, sustainability, efficiency and life cycle of mechanical elements and systems and the requirements related to setting up real-time monitoring systems. Therefore, it is important to emphasize the potential application of deep learning technology to vibration-based condition monitoring of structures and machines [5, 6]. There are several prerequisites when developing sensitive and accurate systems for condition monitoring of machine elements, such as rolling bearings. These prerequisites, which are analyzed in this paper, include an understanding of monitored elements or systems and the development of new, appropriate condition quantification parameters [7].

Because rolling bearings can be regarded as critical elements [8], they have gained significant attention from many researchers in recent years [5, 9, 10], which addressed different problems related to their design, such as contact conditions between rolling elements and raceways [9–11], as well as the development of methods for damage detection.

Although researchers have faced the problem of contact stresses and rolling bearing damage since the beginning of their application, the rapid development of information technologies has caused a significant increase in research focused on this field. Harris [12] gave the analysis of different problems in rolling bearing operation and suggested some causes of rolling bearing damage while Tandon and Choudhury [13] developed an analytical model for vibrational response prediction caused by local damage on rolling bearings. A few years later, they presented an overview [14] of vibro-diagnostic models for damaged rolling bearings. Sopanen and Mikola [15] proposed a dynamic model of rolling bearings with internal clearance and localized damage. This model takes into account Hertz's theory of contact deformations and elasto-hydrodynamic lubrication. It was concluded that the damage on an outer ring causes significantly higher vibration amplitudes than the damage on an inner ring because the outer ring is always in the load zone, and every time the rolling element passes over a damage, the impact impulse appears.

Petersen et al. [16] investigated the influence of damage on changes in stiffness and load distribution. They presented a model for the calculation and analysis of quasi-static load distribution and stiffness variation for radially loaded, two-row rolling bearings with raceway damages of different lengths, depths and roughnesses. The authors concluded that when the rolling element passes over the place of damage, its bearing capacity is reduced or completely lost, and the load is supported by the adjacent rolling elements, which are still running on the undamaged part of the raceway. Petersen et al. [17] also provided an analytical formulation for examining the influence that square damaged areas of different sizes have on the load bearing capacity of the outer ring. The analysis of stiffness variations shows that the mean radial stiffness of the loaded bearing decreases when the damage size increases and increases when the bearing is unloaded.

Malhi [18] worked on improving rolling bearing failure diagnosis. He analyzed the vibrations of rolling elements that pass over a damage on the outer ring of a rolling bearing using finite element method (FEM). Kulkarni and Wadkar [19] analyzed the impact of surface roughness on the vibration level of the outer ring of a radial ball bearing. At constant speed and load, the vibration amplitudes vary with an increase in the magnitude of damage on the outer ring. The experimental results of the obtained vibrational spectrum correspond to the theoretical results. Other recent results [20] present a validation procedure for a single-row ball bearing model that was previously developed by the authors. Experimental tests were performed on bearings with undamaged and damaged outer ring raceways. It was shown that the characteristic bearing frequencies can be easily identified on a physical model. The analysis of experimental results is more complex due to the presence of certain vibrations that arise from installation components. A comparison of the characteristic frequencies obtained by the model in relation to the experimental and theoretical results was made, and it showed very good correlation. Leturiondo et al. [21] proposed a methodology for the physical modeling of ball bearings, showing that the obtained vibration spectrum is not only an indicator of the presence of damage but also provides information about its type. In this paper, simulations of two types of bearings (ball bearing REXNORD ER16K and cylindrical roller bearing NJ 305) are presented. The influence of the damage on their dynamic behavior was analyzed, with the results corresponding with theoretical and experimental results obtained by other authors.

Recently, Shaikh and Kulkarni [22] have developed a theoretical model of a ball bearing movement with two degrees of freedom for vibration prediction and frequency response analysis. The contact between the rolling elements and raceway was considered as being similar to a nonlinear spring. Contact forces were obtained by Hertz's theory of contact deformations. Nonlinear second-order differential equations were solved by MATLAB software. The impact of the speed and magnitude of damage on the outer ring raceway showed changes in the frequency spectrum. It is evident from the frequency spectrum that an increase in the speed of an undamaged bearing causes an increase in the vibration amplitude. However, the value of the vibration amplitude of an undamaged bearing is lower than the amplitude of a damaged bearing for the same speed range. Furthermore, it has been shown that the vibration amplitude increases with an increase in the defect size.

Based on the presented discussions, as well as on the series of up-to-date research [23–27], it is obvious that the use of a simple and effective bearing condition monitoring system is essential in state-of-the-art machinery and industry. A systematized unified approach, and effective bearing damage indicator, is thus needed.

Despite its importance to the industry and the many related works, it seems that a systematized approach for condition monitoring of bearings does not yet exist. With this purpose in mind, a methodology for analyzing radial ball bearings based on previous work by the same authors [28] is presented in this paper. This methodology uses a parameter for rapid damage assessment that could also be utilized in a computerized condition monitoring system.

## 2 Introduction of a Damage Factor Calculation for Condition Monitoring

A new parameter named the *damage factor* is introduced to quantify the vibration response of rolling bearings with damaged contact surfaces. The calculation of this parameter is recognized as the main objective of a mathematical model real-time condition monitoring of rolling bearing nonlinear vibrations, in accordance with the demands for digitalization of the manufacturing processes. A schematic representation of such a system is shown in Fig. 1.

Considering the requirements for being a simple and reliable parameter for prediction and estimation of operational condition of any rolling bearing, this new parameter is defined as follows: The damage factor is the ratio of the absolute value of the maximum vibration amplitude (measured or calculated as acceleration) of a bearing with damage to the absolute value of the vibration amplitude (measured or calculated as acceleration) of an undamaged bearing for the same loading conditions:

$$K_V = \frac{A_d}{A_u} \qquad (1)$$

where $A_d$ is the maximum vibration amplitude of a damaged bearing [m/s$^2$] and $A_u$ is the vibration amplitude of an undamaged bearing [m/s$^2$].

The damage factor $K_V$ defined by relation (1) depends on the main parameters that affect rolling bearing vibrations, such as the applied load value, bearing type and dimensions, as well as form of lubrication and extension of the damage. This parameter can also be explained as a quantitative measure of the influence that predefined damage parameters have on certain bearing performance characteristics, such as structural vibrations, and was adopted to provide a measure for the bearing condition.

### 2.1 Review of Damage Factor Applicability

Recently published papers presented many analytical, experimental and numerical results that can be used to validate the approach proposed in the present work for estimating bearing conditions.

- Patel et al. [29] performed experimental testing of radial ball bearing vibrations with damage on inner and outer ring raceways. For this purpose, the SKF BB1 B420205 bearing was selected. Damage on the inner ring raceway had the shape of a hemisphere with diameters of 0.42, 0.5, 0.96 and 1.48 mm. At the specific frequency of the bearing's inner ring, vibration amplitudes were measured for the corresponding damage dimensions, and the following values were obtained: 0.1, 0.16, 0.22 and 0.42 mm/s$^2$. Additionally, at the specific frequency of the outer ring, vibration amplitudes of 0.25, 0.3, 0.6 and 0.74 mm/s$^2$ were measured for the same radial load. The peaks of vibration amplitudes from diagrams presented in this work clearly show the existence of damage. Thus, it could be concluded that the vibration amplitudes increase with an increase in the damage magnitude. By dividing the values of the vibration

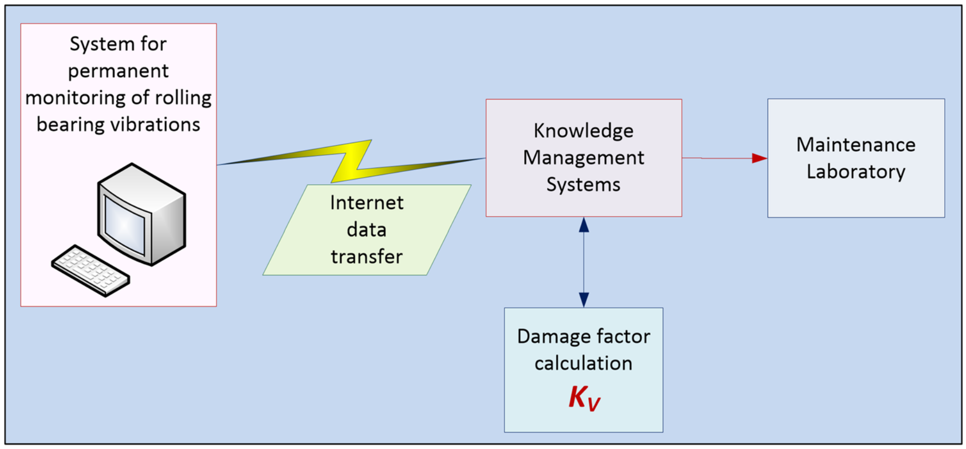



**Fig. 1** Schematic representation of a system for real-time rolling bearing condition monitoring based on digitalization principles

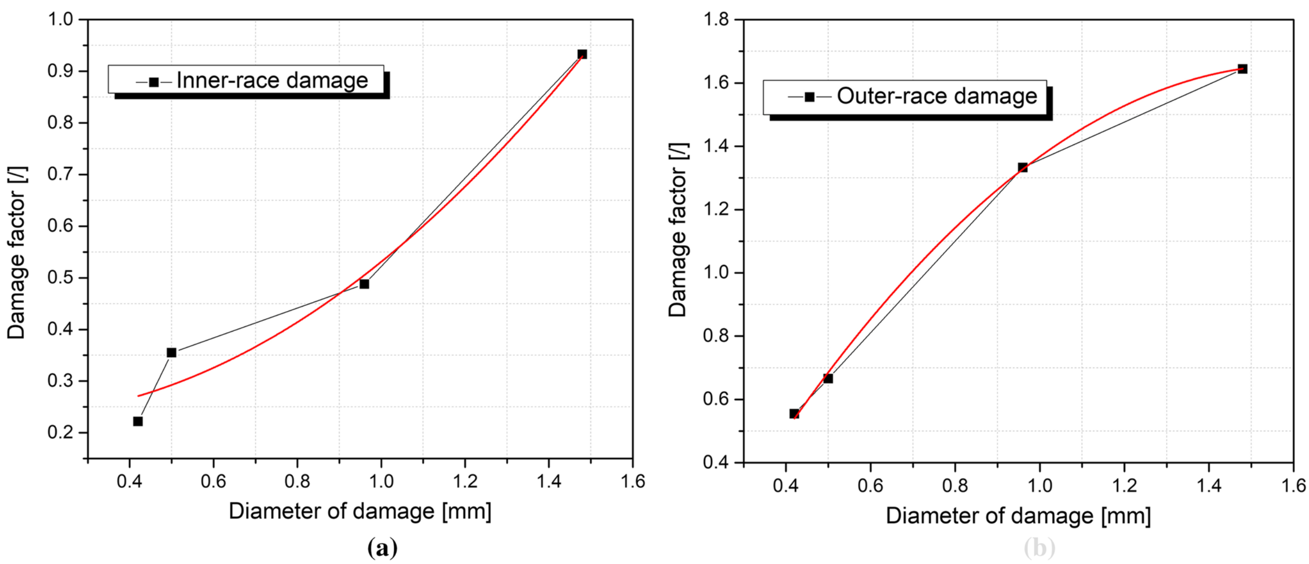



**Fig. 2** Damage factor variation for bearing SKF BB1 B420205 for different damage dimensions on the inner raceway (**a**) and outer raceway (**b**)

amplitudes obtained for damaged bearing samples by those obtained for undamaged bearing samples, the damage factor proposed herein can be calculated and represented as shown in Fig. 2.

- Kumbhar et al. [30] investigated the experimental testing of tapered roller bearings SKF 30,208 with damages of circular and square shape on the outer ring raceway and on the rolling elements. During the experiment, the load and the speed of rotation were varied for different dimensions and forms of damage. Loads of 500, 1000 and 1500 N were applied at 800, 1100 and 1400 rpm. The obtained results showed that with an increase in the speed of rotation, an increase in the damage dimensions causes an increase in the vibration amplitude, while a decrease in the load causes an increase in the amplitude of vibration. Furthermore, the change in the bearing damage shape causes an increase in the value of the vibration amplitude value. However, the influence of the damage size is considered substantially greater and more significant. Based on the obtained vibration amplitudes for different bearing damage dimensions and for different loads, a diagram representing the damage factor dependence on the given parameters was drawn, as shown in Fig. 3. The objective was to determine how the impact factor was affected by the mentioned parameters.
- Unlike previous authors, Kulkarni et al. [31] dealt with numerical calculations to verify their own results. Bearing 6205 with damage dimensions of 0.5, 1 and 1.5 mm on the outer raceway was analyzed. During the calculations, the speed of rotation was set to 2400 rpm, while the external radial load varied and was set equal to 212, 424, 636 and 848 N. The obtained results of vibration amplitude for different damage dimensions at the same load of 424 N reconfirmed that an increase in damage dimensions causes an increase in the vibration amplitude. This result was confirmed by the experimental results shown in Fig. 4a. It was determined that an increase in the applied load intensity

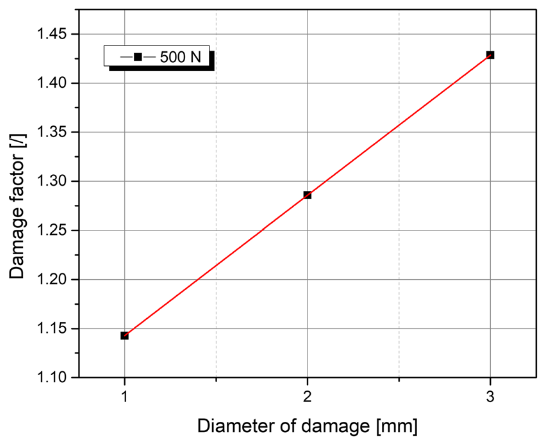
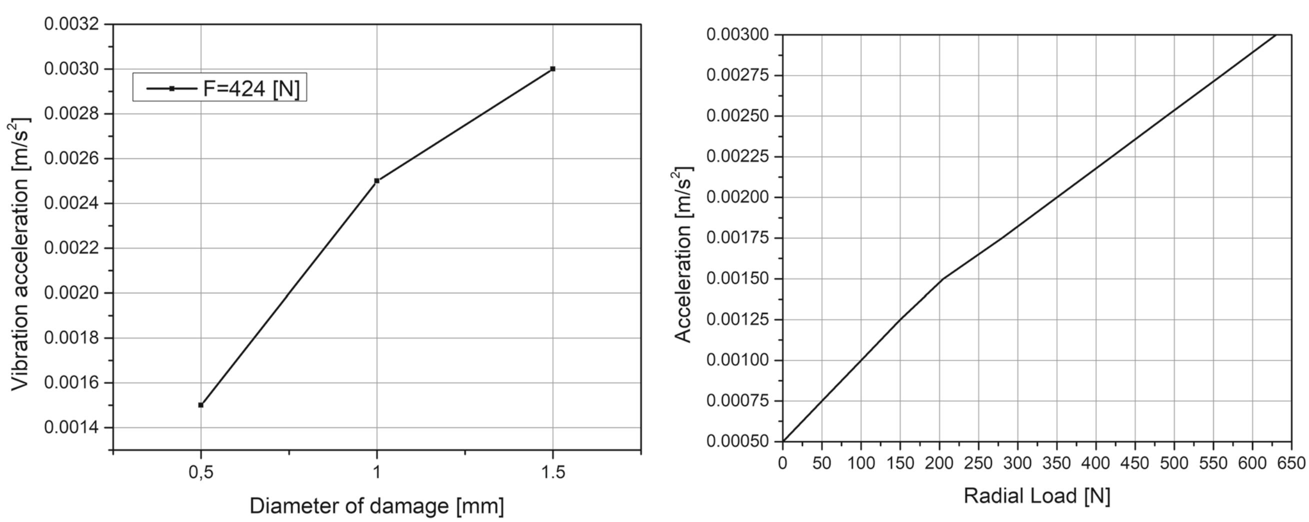



**Fig. 3** Damage factor variation for different damage dimensions on the outer raceway for bearing SKF 30,208 for a loading of 500 N at 800 rpm

for a certain damage dimension will consequently cause a significant increase in the vibration level amplitude. This increase takes the shape of characteristic peaks that occur at a specific frequency of the outer bearing ring, which is 144 Hz, as shown in Fig. 4b. Due to the absence of data for the vibration amplitude of undamaged bearings, calculation of the damage factor was not possible.

- Ding et al. [32] developed a 3D finite element model for vibrational response estimation for 6205 rolling bearings with damage on the outer raceway using the Abaqus software package. Three different damage lengths (1.98 mm, 3.95 mm and 5.93 mm) were analyzed, while the radial depth was 0.1 mm. It has been shown that the stiffness of the model decreases exactly when the rolling element enters the damage zone, whereby a shock wave is generated. It was also concluded that the impact of bearing damage is directly related to the magnitude of damage because the increase in damage size causes an increase in the vibration amplitude. The published results of these authors for vibration amplitude were utilized in our research to calculate the damage factors for different damages, thereby determining the applicable trend, as shown in Fig. 5.
- Kulkarni and Bewoor [33] conducted a theoretical analysis to obtain the vibration response of rolling bearings with a single extended distributed type of damage on the outer raceway. The response of the derived model was validated with the experimental results under constant radial load. The observation was noted under the same load and speed conditions, i.e., at 1000 N radial loading at 1200 rpm, while damage sizes of 10, 20, 30 and 40 microns were considered. The conclusion may be drawn from the theoretical analysis that a damage size has a major impact on the vibration response of a bearing, as shown in Fig. 6. The model also predicts the peaks of the frequency spectrum observed at the outer ring frequency, its harmonics and a combination of cage frequency and shaft frequency.

The analysis presented above, based on the published analytical and experimental results from other researchers, shows that the damage factor $K_V$ introduced by Eq. (1) obtains similar functions depending on the damage size in all of the analyzed cases.




**(a)** **(b)**

**Fig. 4** Vibration amplitude variation in the 6205 bearing for an applied loading of 424 N at 2400 rpm: **a** for different damage dimensions on the outer raceway; **b** for different applied loads

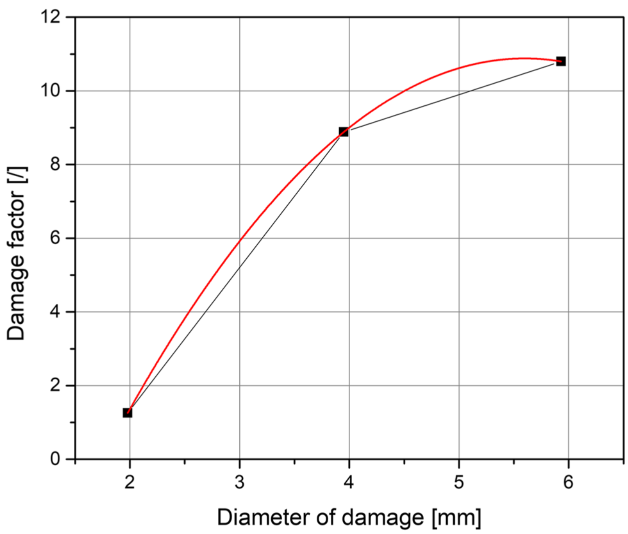



**Fig. 5** Damage factor diagram for different damage dimensions on the outer raceway of 6205 bearings

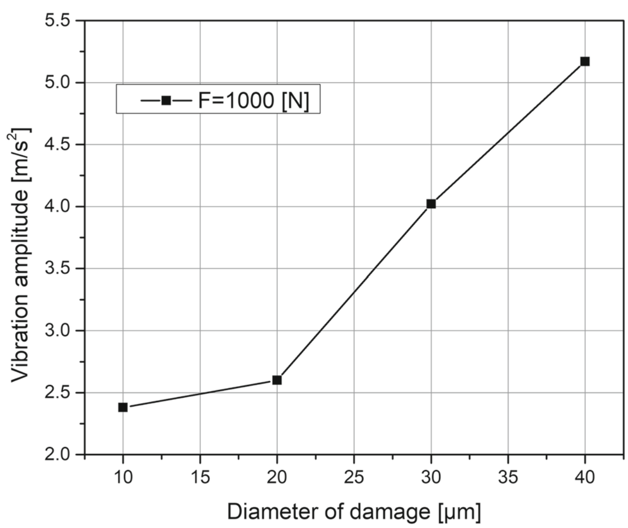



**Fig. 6** Bearing vibration amplitude at 1000 N radial loading at 1200 rpm for different damage sizes

## 3 Analytical–Numerical Method for Damage Factor Calculation

The damage factor $K_V$ can be assessed experimentally or by an analytical–numerical method, as presented in Fig. 7. The developed analytical–numerical method uses available computational numerical methods to provide an approximate solution for differential equations of motion (Runge–Kutta method) and stress–strain state calculations (finite element method). This method can be employed for developing computer programs for simple, rapid and unified calculations of damage factor functions for a particular type of rolling bearing. The input in this algorithm consists of the main bearing parameters, while the output is given in the form of $K_V$ functions, which are required for programming artificial intelligence systems for real-time monitoring.

### 3.1 Damaged Ball Bearing Parameter Calculation

To determine the load distribution and consequent bearing deformation [34], the developed FE model referred to in Fig. 7 should be based on multi-body contact in nonlinear dynamics [35].

For the concept presented in this paper, a particular radial deep groove ball bearing type (SKF 6206) was chosen for testing and experimental validation of the developed method for damage factor calculation. The main characteristics and dimensions for this rolling bearing are given in Table 1. In accordance with the dimensions and characteristics of the SKF 6206 bearing, a 3D FE model was created using ANSYS, as shown in Fig. 8. Detailed descriptions and discussions on developing an FE model and carrying out a finite element analysis (FEA) of undamaged and damaged bearings of this type were recently presented by Atanasovska, Soldat et al. [28, 34, 36].

Finite element analysis (FEA) was conducted for different values of external load within the load capacity of the bearing, as given in Table 1. In order to make numerical simulation to be low computer time consuming, the ideally lubrication was assumed and fluid–solid interaction was neglected in the finite element models. The quasi-static conditions are used in order to determine the system stiffness parameter. The contact zones are modeled with a friction coefficient of 0.3 [37, 38], corresponding to the lubricated materials in contact (Table 1). The results obtained for contact deformations and load distributions on the bearing elements were collected from this analysis and used for calculations of the parameters and factors required in the following steps of the developed method. The results for the radial deformation of the 6206 bearing for a cone-shaped damage with a diameter of 2 mm under a 2000 N external load are presented in Fig. 9.

Additionally, finite element analyses provide maximum stresses on bearing elements, which constitutes important data for deep learning monitoring systems. In this regard, this system can be utilized to define a critical damage factor value with respect to potential overloads in contact zones with damages and bearing failure. A command to decrease the applied load can be generated, thus avoiding catastrophic failures.

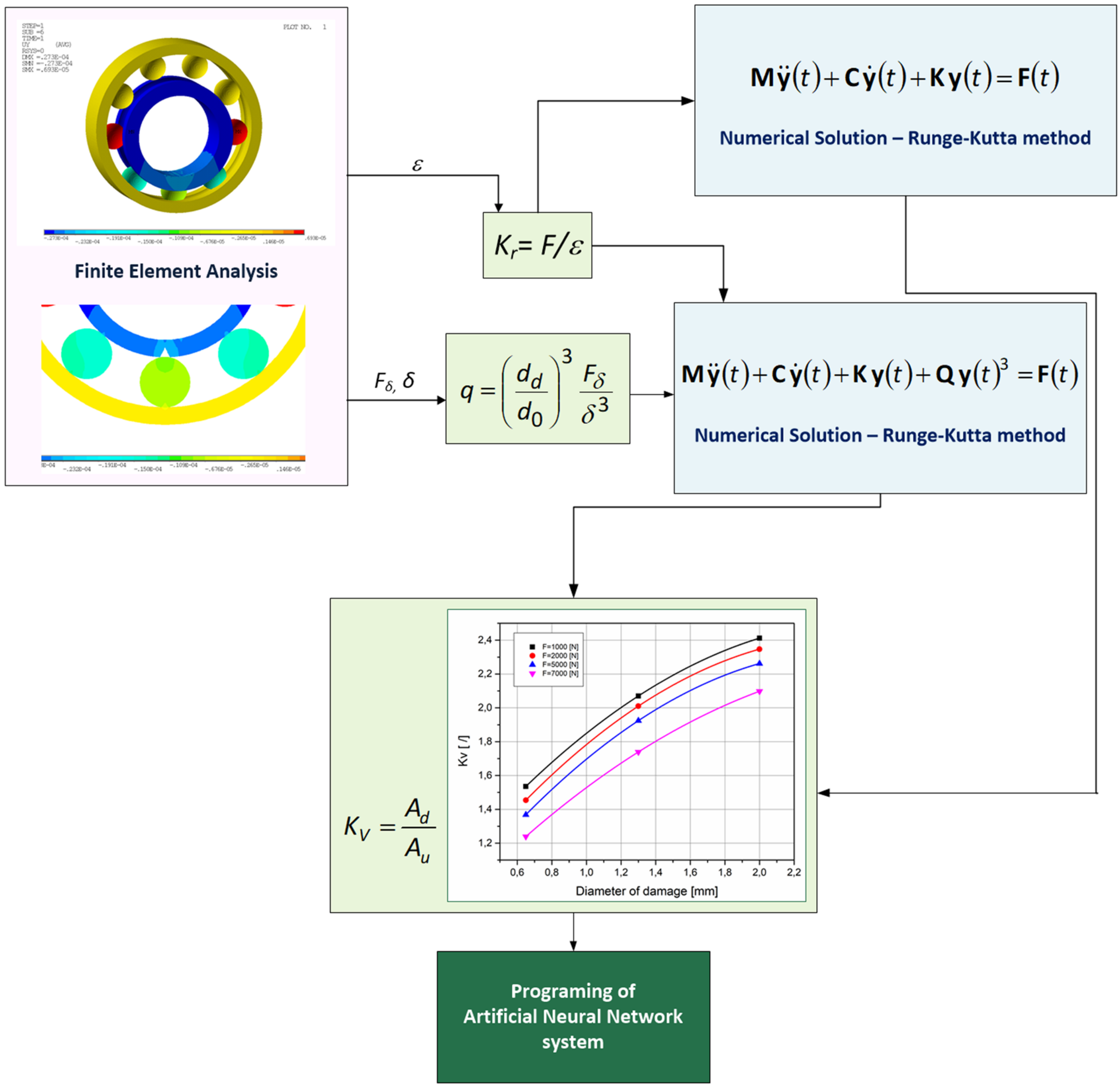



**Fig. 7** Algorithm of a new analytical–numerical method for damage factor calculation

## 4 Analytical–Numerical Model for Damaged Rolling Bearing Dynamics

The following system of differential equations of motion describes rolling bearing dynamics in general [28, 39, 40]:

$$\mathbf{M}\ddot{\mathbf{y}}(t) + \mathbf{C}\dot{\mathbf{y}}(t) + \mathbf{K}\mathbf{y}(t) = \mathbf{F}(t) \tag{2}$$

where $\mathbf{M}(t)$ is the mass matrix, $\mathbf{C}(t)$ is the damping matrix, $\mathbf{K}(t)$ is the stiffness matrix, and $\mathbf{y}(t)$ and $\mathbf{F}(t)$ are the vectors of the generalized displacements and external forces.

For radial deep groove ball bearings, this model can be reduced to a one-degree-of-freedom system, as described by Atanasovska [40]. Thus, the differential equation of motion for the analytical–numerical calculation of vibration amplitudes for undamaged radial rolling bearings has the following form:

$$m_{red}\,\ddot{y} + C_r\,\dot{y} + K_r\,y = F \tag{3}$$

where $m_{red}$ is the reduced mass of system shaft-bearing-casing; $C_r$ the radial ball bearing damping; $K_r$ the radial ball bearing stiffness; and F an external load.

**Table 1** Main characteristics of the analyzed rolling bearing

| Parameter | Symbol and units | Value |
|---|---|---|
| *Radial deep groove ball bearing SKF 6206* | | |
| Outer race diameter | $D$ [mm] | 62 |
| Inner race diameter | $d$ [mm] | 30 |
| Bearing width | $B$ [mm] | 16 |
| Number of rolling elements | $z$ | 9 |
| Diameter of rolling element | $d_b$ [mm] | 9.525 |
| Angular distance between rolling elements | $\gamma$ [degrees] | 40 |
| Diameter of the inner race | $d_i$ [mm] | 36.475 |
| Diameter of the outer race | $d_o$ [mm] | 55.525 |
| Inner ring groove radius | $r_{ti}$ [mm] | 4.86 |
| Outer ring groove radius | $r_{to}$ [mm] | 5.05 |
| Static load capacity | $C_0$ [N] | 11,200 |
| Dynamic load capacity | $C$ [N] | 19,500 |
| Young's modulus | $E$ [N/m$^2$] | $2.06 \cdot 10^{11}$ |
| Poisson's ratio | $\mu m$ | 0.3 |

The total radial deep groove ball bearing stiffness $K_r$ is a ratio between the external load and generated total deformation [36, 40, 41], given as:

$$K_r(y, \mu, t) = F/\varepsilon(y, \mu, t) \quad (4)$$

where $F$ is the constant external radial load; $\varepsilon$ $(y, \mu, t)$ is the elastic displacement of the bearing axis, which is equal to the total deformation of the ball bearing in the radial direction, i.e., the total radial displacement of the axis of the bearing in the radial direction obtained by FE analyses; $\mu$ is the friction coefficient in contact zones; and $t$ is time.

For the chosen undamaged radial ball bearing, FE analyses were performed, and the total radial stiffness was calculated. The influence of a lubrication was disregarded in the FE analyses, in accordance with the models verified by other authors [37, 38, 42]. However, it is additionally considered in the Eq. (3) by a damping constant $C_r$ [25] that can be assumed to be proportional to the corresponding stiffness parameter [43]. The vibrational acceleration amplitudes calculated by Runge–Kutta method [28] are given in Table 2.

A mathematical model for the dynamics of damaged rolling bearings (rolling bearings with surface damage on the inner or outer raceway) is developed based on the postulates of nonlinear dynamics [44–46]. The differential equation of motion for a damaged rolling bearing is defined as a special case of a general nonlinear oscillator [45] with strong cubic nonlinearity, expressed with a nonlinearity coefficient [44]:

$$\mathbf{M}\ddot{\mathbf{y}}(t) + \mathbf{C}\dot{\mathbf{y}}(t) + \mathbf{K}\mathbf{y}(t) + \mathbf{Q}\mathbf{y}(t)^3 = \mathbf{F}(t) \quad (5)$$

where $\mathbf{Q}(t)$ is a vector of the nonlinearity coefficient, which is defined to be dimensionally consistent with Eq. (5) and to be dependent on the damage dimension, local contact deformation and bearing load distribution.

For radial deep groove ball bearings, Eq. (5) can be reduced to a one-degree-of-freedom system:

$$m_{red}\ddot{y} + C_r\dot{y} + K_r y + q y^3 = F \quad (6)$$

The nonlinear part of this equation, represented by the nonlinearity coefficient $q$, shows how existing damage influences the rolling bearing vibrations. For a particular radial rolling bearing, this differential equation is solved by the numerical Runge–Kutta method and *MATLAB* software, as shown in Fig. 7. The obtained vibration amplitudes depending on the external load and damage dimension are shown in Fig. 10.

A formula for the calculation of the nonlinearity coefficient $q$ is obtained by an iterative procedure based on the experimental measurements of the bearing vibration

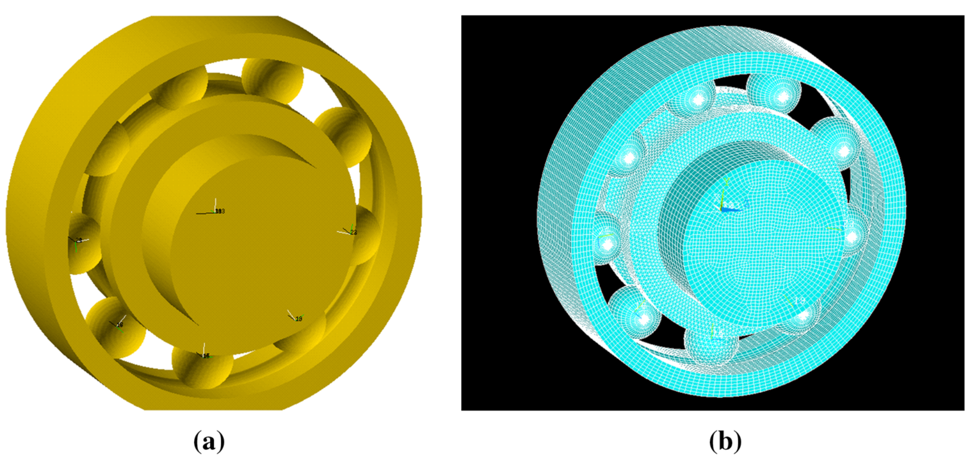
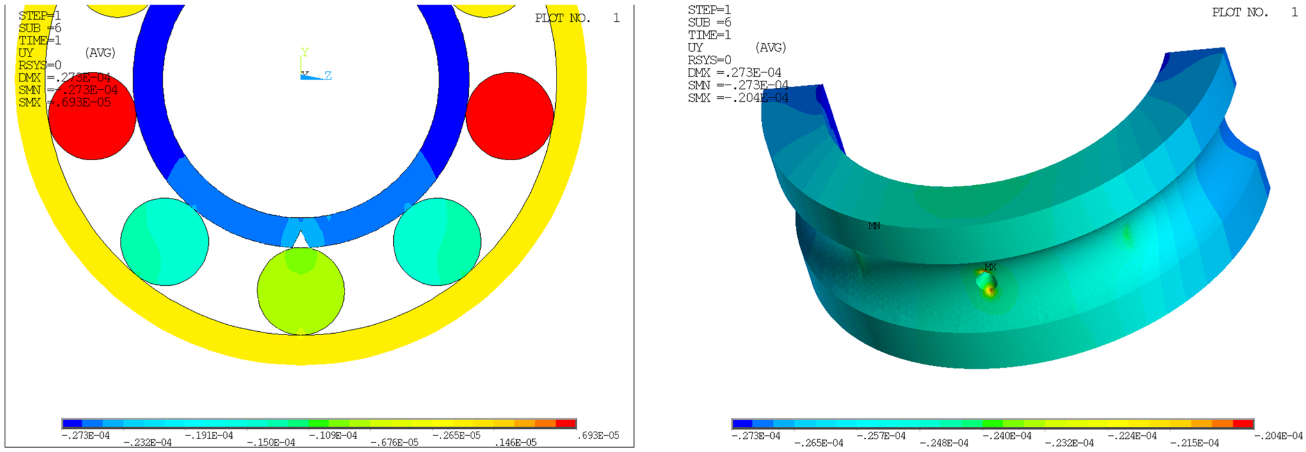


**Fig. 8** Finite element model of radial ball bearing SKF 6206: **a** 3D model **b** meshed model




**(a)** **(b)**

**Fig. 9** Contour plot of radial deformation obtained by FE analyses for the 6206 bearing: **a** cross section of the 3D bearing model; **b** view of the damaged surface of the inner raceway

response. The nonlinearity coefficient depends on the damage dimension, local contact deformation and external load distribution among rolling elements. Based on an empirical iterative procedure that is performed to obtain a formula with minimal deviation from the experimental results, the following formula is proposed for the 6206 bearing:

$$q = \left(\frac{d_d}{d_0}\right)^3 \frac{F_\delta}{\delta^3} \tag{7}$$

where $d_d$ is the diameter of the damage on a contact surface; $d_0$ is the average diameter of the damage for a particular rolling bearing type obtained by the producer; $F_\delta$ is a load distributed on the most loaded rolling element, obtained by FEA; and $\delta$ is a local deformation on the most loaded rolling element in contact with the damaged raceway surface, obtained by FEA.

The developed method was applied to the radial deep groove ball bearing SKF 6206 for cone-shaped damages on the inner raceway with three different damage diameters of 0.65 mm, 1.3 mm and 2.0 mm. Figures 11 and 12 show the obtained values for the nonlinearity coefficient and damage factor, respectively.

The damage factor can be described by a second-order polynomial function from the damage diameter based on the analysis of the obtained bearing vibration amplitudes for undamaged and damaged SKF 6206 bearings. This function shows less than 3% deviation from the calculated data for different damage diameters:

$$K_V(x) = A + B_1 d_d + B_2 {d_d}^2 \tag{8}$$

where $A$, $B_1$ and $B_2$ are constants.

This function for the analyzed bearing type has the following form related to different external loads:

$$K_V(1000N) = 0.79062 + 1.30625 d_d - 0.24778 {d_d}^2 \tag{9}$$

$$K_V(2000N) = 0.66393 + 1.39554 d_d - 0.277 {d_d}^2 \tag{10}$$

$$K_V(5000N) = 0.57883 + 1.39348 d_d - 0.275958 {d_d}^2 \tag{11}$$

$$K_V(7000N) = 0.57853 + 1.13955 d_d - 0.18991 {d_d}^2 \tag{12}$$

The analysis of the obtained polynomial functions, as drawn in Fig. 12, shows that an increase in the damage diameter can cause an increase in the damage factor, which actually means an increase in vibration amplitude.

## 5 Experimental Validation

Experimental testing was performed to determine vibration amplitudes, as presented in Fig. 7, which consequently verifies the mathematical model of bearing dynamics, as well as two newly proposed parameters, $K_v$ and $q$. An extensive

**Table 2** Maximum vibration amplitudes for undamaged ball bearing SKF 6206 [28]

| F [N] | 1000 | 2000 | 5000 | 7000 |
|---|---|---|---|---|
| Vibration amplitudes [m/s$^2$] | 2000 | 4000 | 10,000 | 14,000 |

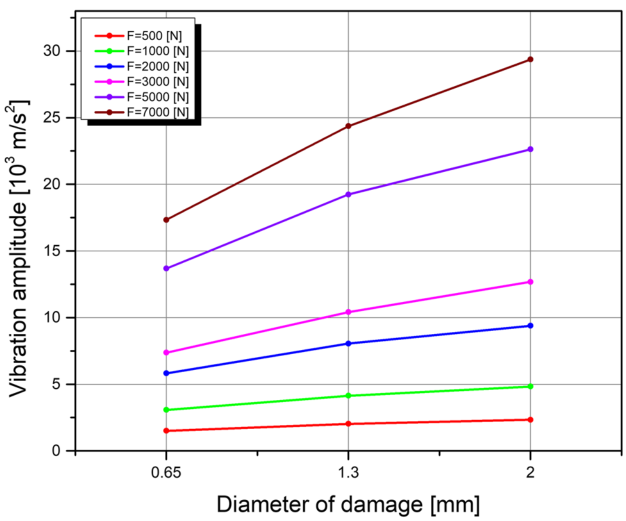



**Fig. 10** Vibration amplitudes for the 6206 bearing

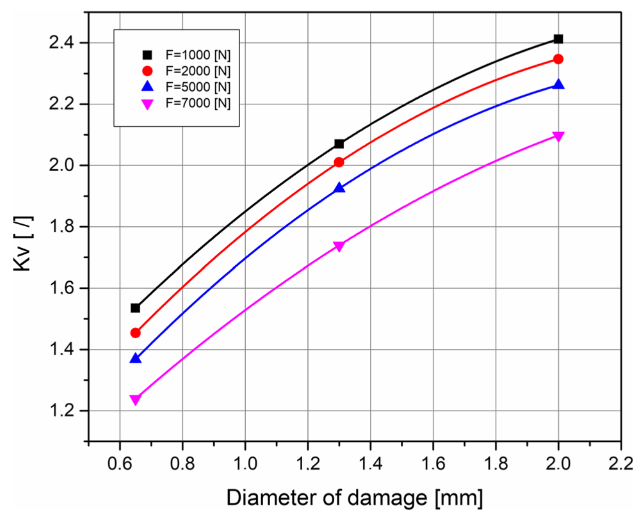



**Fig. 12** Damage factor variation for the 6206 bearing depending on the total external load and damage dimension

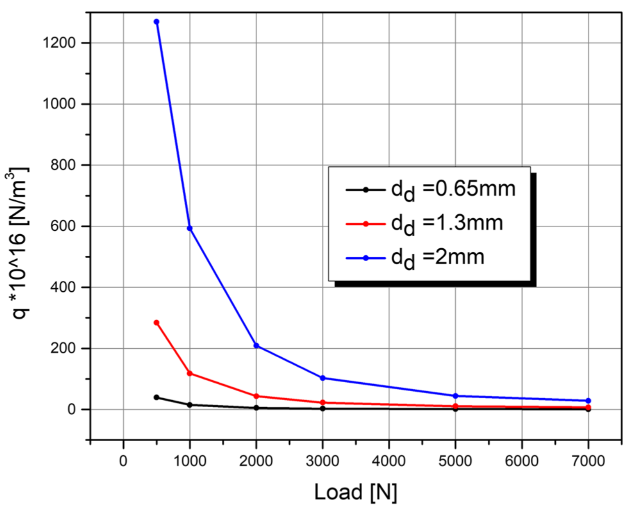



**Fig. 11** Nonlinearity coefficient variation for the 6206 bearing depending on the total external load and damage dimension

number of samples of the 6206 bearing were produced, especially for this experiment (Fig. 13). The conical damages on the inner raceways are made before heat treating and assembling. A control measurement of the damage dimensions was taken by a HIROX KH-7700 digital 3D microscope with an optical zoom of 50 × to 400x, as presented in Fig. 14. The experiment was carried out on the dynamic test ring developed and explained by Soldat et al. [33]. Absolute vibration was measured on the outer ring during gradual variation in the external radial load until the static load bearing capacity was reached. A mounted sensor measures vibrations in the radial direction at a constant shaft angular velocity of 1476 rpm, i.e., $\omega = 154.5\ \mathrm{s}^{-1}$.

The value of the specific inner ring frequency $f_i$ is calculated for $d_b/d_c = 0.207$ in accordance with the following equation [33]:

$$f_i = \frac{z}{2}\omega\left(1 + \frac{d_b}{d_c}cos\alpha\right) \qquad (13)$$

where $\omega$ is the angular rotor velocity; $z$ is the total number of rolling elements; $d_b$ is the diameter of the rolling elements; $d_c$ is the diameter of the dividing axis of the bearing cage; $d_b/d_c = 0.207$; and $\alpha$ is the bearing contact angle, which is zero for the SKF 6206 bearing [28].

Recorded vibrations were obtained in the time domain and transformed into the frequency domain by a developed application [28]. In accordance with the calculated inner ring frequency of $f_i = 839$ Hz, the characteristic peaks are read, as shown in Fig. 15.

The results obtained analytically and experimentally are depicted in Fig. 16 by means of a comparative diagram which shows a high level of superposition and validation of the developed analytical–numerical method. The maximum difference is about 7%.

## 6 Discussion and Conclusions

The research of rolling bearing behavior presented in this paper is inspired by the new trends in machineries and industries. The permanent monitoring of critical parts, such as rolling bearings, is necessary for achieving increased reliability, safety and energy efficiency, as well as operation with minimized probability of failure. For these purposes, a new approach and parameters were developed and discussed in this paper. The presented analytical–numerical and experimental research leads to the following conclusions:

**Fig. 13** Manufactured 6206 inner ring bearing samples

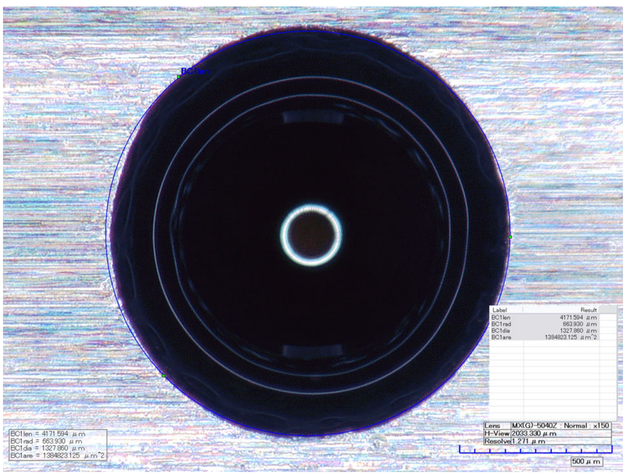


**Fig. 14** Damage diameter dimension measuring by digital 3D microscope

- In accordance with contemporary demands, the development of new quantification parameters for the condition monitoring of rolling bearings is required. A new damage factor is introduced, and a universal approach for its calculation is developed and described by an algorithm.
- For a particular type of radial ball bearing, an investigation of the applicability of the damage factor was performed.
- The obtained results are discussed, and a trend of increasing bearing vibration is obtained when the damage dimensions increase and when the external load increases. This finding validated the suitability of measuring vibration amplitudes for damage assessment on rolling bearings .
- A damage factor is calculated for different external loads for a particular rolling bearing, and an appropriate polynomial function is selected to present how this parameter depends on the damage dimensions. This function showed deviations less than 3% from the calculated damage factor data for different damage dimensions.
- A comparison of the experimentally obtained results and those obtained by the developed method shows a high level of superposition, with deviations up to 7%.
- The applicability of the developed approach in a general system for real-time rolling bearing condition monitoring based on the application of deep learning or other contemporary information technologies was discussed. Such a system will enable a closed-loop connection of measured vibrations with a predefined critical damage factor value in accordance with an algorithm for the numerical calculation of the stress–strain state on bearing elements. In this way, the developed system could ensure the operation of rotating systems and machinery without failure due to bearing malfunction. The computer programing system based on machine learning could also be suitable when assessing the need for bearing replacement or loading reduction to avoid system failure.

The main contribution of the presented research is an introduction of a new approach for solving the described problem. Further research will consider more influencing factors, such as mathematical modeling of bearing lubrication with variable parameters, as well as different shapes and dimensions of damages.

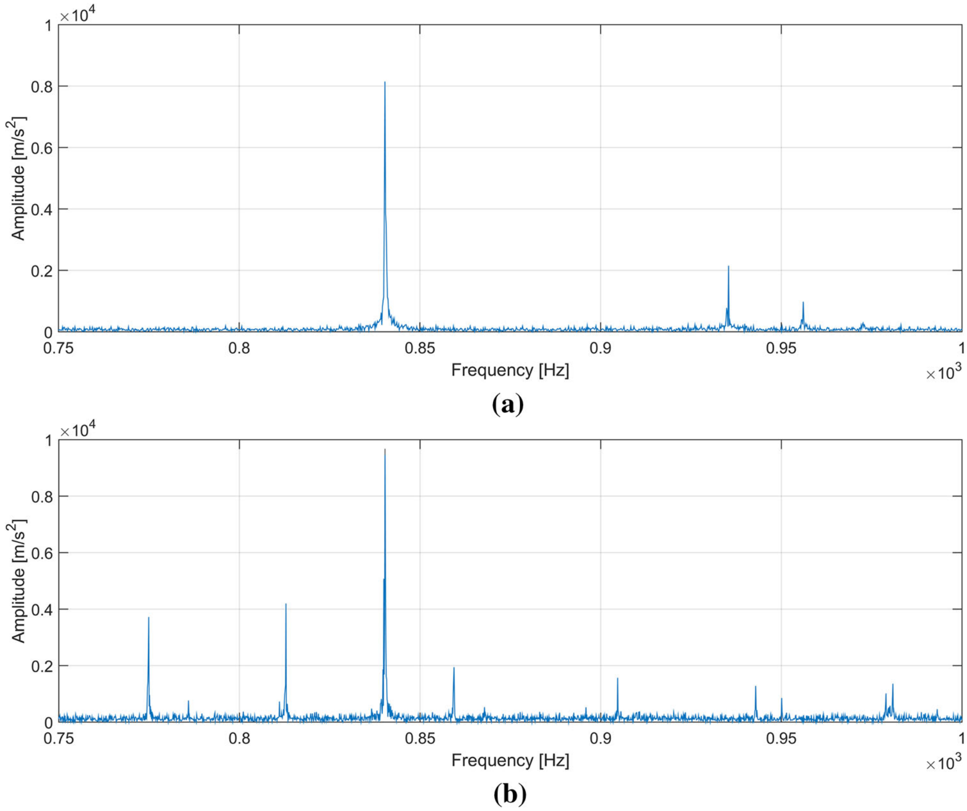



**Fig. 15** Examples of experimentally obtained results for the 6206 bearing for 2000 N external loading and damage diameters of **a** 1.3 mm and **b** 2.0 mm

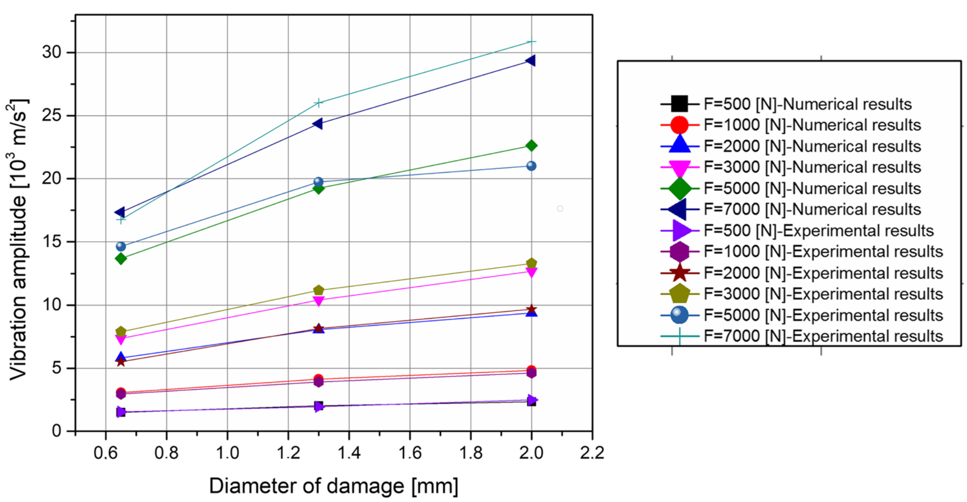



**Fig. 16** Comparative diagrams for vibration amplitudes of damaged bearing 6206, obtained analytically and experimentally

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